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STUDY OF DIFFERENT TYPES OF CONNECTION IN VARIOUS DIFFERENTIABLE MANIFOLD

VikashBhatt[1], Kamal Kishor[2], Dr. Shankar Kumar[3]
[1]PhD Scholar, [2]PhD Scholar, [3]Assistant Professor, Mathematics Department, Soban Singh Jeena University, Almora, Uttarakhand 263601, India
Corresponding Author: Vikash Bhatt: E-mail: *vikashbhatt48@gmail.com

Abstract: *The present work investigates numerous types of affine connections defined on differentiable manifolds, with a focus on Lorentzian para-Sasakian (LP-Sasakian) manifolds with non-Levi-Civita structures. We construct explicit equations for the related torsion, curvature, Ricci tensor, and scalar curvature by adding a semi-symmetric non-metric connection that is consistent with the underlying LP-Sasakian structure, illuminating their differences from traditional Riemannian equivalents. The paper rigorously describes generalized pseudo Ricci symmetric, generalized Ricci-recurrent, semi-pseudo symmetric, and semi-pseudo Ricci symmetric manifolds using covariant differential constraints on curvature and Ricci tensors. Several structural theorems are established utilizing tensorial identities, Bianchi-type relations, and contraction techniques, revealing that generalized Ricci-recurrent LP-Sasakian manifolds admitting Codazzi or cyclic type Ricci tensors necessarily reduce to Einstein manifolds under the considered connection.Furthermore, the non-existence of semi-pseudo symmetric and semi-pseudo Ricci symmetric LP-Sasakian manifolds (for dimension $n>3n>3$) permitting a semi-symmetric non-metric connection is convincingly shown, underlining inherent geometric impediments. The provided conclusions not only unify several ideas of curvature symmetry within a generalized affine framework, but they also provide a better understanding of how non-metricity and torsion impact the global geometric behavior of differentiable manifolds. This study adds to the larger theory of non-Riemannian geometry and establishes a foundation for future research in extended geometric structures and mathematical physics.*

Keywords: Differentiable manifold, connections, Sub-manifolds, LP-Sasakian

1. Introduction:

For all mathematicians, regardless of their areas of expertise, differential geometry is a fundamental topic since it provides the fundamental concepts and instruments required by physicists and engineers. Differential geometry is perhaps the oldest field of mathematics, and it was developed far after calculus was established by Newton and Leibnitz. Geometric spaces and forms that may be described by differentiable functions are studied in differential geometry. Differentiable manifolds are mathematical objects that are studied. Differentiable manifolds have historically appeared in a wide range of mathematical fields. Examples that served as the foundation for the notion of differentiable manifold included local differential geometry, projective geometry, algebraic geometry, Riemannian surface theory, and continuous or Lie groups. The differential geometric features of submanifolds of manifolds with certain structures are broad and productive areas of Riemannian geometry, and the theory of structure on manifolds is an intriguing issue in contemporary differential geometry.

One of the fascinating subjects in differential geometry is the theory of submanifolds of a nearly Hermitian manifold. A vector in an almost Hermitian manifold is transformed into a vector perpendicular to it by its almost complex structure F. The behavior of the tangent bundle of submanifolds of nearly Hermitian manifolds under the action of the ambient manifold's nearly complex structure F may have served as a natural incentive to investigate these submanifolds. Invariant submanifolds and anti-invariant submanifolds are the two recognized classes of submanifolds. The tangent space of the submanifold is mapped into the normal space in the second scenario, whereas it is invariant under the action of the nearly complex structure F in the first.

In 1978, Bejancu began studying differential geometry of submanifolds as a generalization of invariant and anti-invariant submanifolds of an essentially Hermitian manifold. Several geometers followed suit. Semi-invariant submanifolds in Sasakian manifolds were investigated by Bejancu and Papaghuic in 1981. Under the term contact CR-submanifold, Kobayashi (1981) and Yano-Kon (1982, 1983) investigated the similar idea.

The idea of a CR-submanifold of a Kaehler manifold is simply extended to submanifolds of virtually contact metric manifolds to create semi-invariant submanifolds. Semi-invariant submanifolds of a certain class of nearly contact manifolds were examined by Kobayashi (1986). The trans Sasakian structure is a novel class of almost contact structures that Oubina (1985) developed. Some results about a nearly semi-invariant submanifold of an SP-Sasakian manifold were provided by Kalpana (1989). CR-submanifolds of a trans Sasakian manifold were examined by Shahid (1991).

Additional results on CR-submanifolds of a trans Sasakian manifold were provided by Shahid (1994). Semi-invariant submanifolds of a Kenmotsu manifold with constant ~-holomorphic sectional curvature were investigated by Sinha (1992) and Srivastava. Semi-invariant submanifolds of a trans Sasakian manifold were examined by Tripathi (1996). The CR-submanifold of a nearly and tightly cosymplectic manifold was examined by Tripathi (2000). Bhatt and Dube (2002) investigated semi-invariant submanifolds of nearly para r-contact manifolds, semi-invariant submanifolds of nearly r-Sasakian manifolds, and CR-submanifolds of nearly and tightly para Cosymplectic manifolds. Semi-invariant submanifolds of Sasakian space form were investigated by Joshi and Dube (2003).

Semi-invariant submanifolds of r-Kenmotsu manifolds were examined by Bhatt and Dube (2003). Nearly hyperbolic Hermitian manifolds were investigated by Dube (1973). An nearly r-contact hyperbolic structure in a product manifold was examined by Dube and Niwas (1978). Hypersurfaces of nearly hyperbolic Hermite manifolds were examined by Pal and Mishra (1980). Hypersurfaces submerged in an essentially hyperbolic Hermitian manifold were examined by D~be and Mishra (1981). Hypersurfaces of nearly Hermite manifolds were examined by Mishra (1993). A semi-invariant submanifold of an approximately r-contact hyperbolic metric manifold was examined by Joshi and Dube (2001). Bhatt (2002) investigated nearly hyperbolic Hermitian manifolds' hypersurfaces.

A specific transformation on Lorentzian para Sasakian manifolds was researched by Matsumoto (1988). He conducted more research on Lorentzian paracontact manifolds in 1989. In 1994, Prasad, Shanteshwar, and Ojha conducted research on Lorentzian para contact submanifolds. In a Lorentzian paracontact manifold, Pandey and Ojha (2001) investigated a semi-symmetric metric and non-metric connection. A semi-symmetric non-metric $-connection in a Lorentzian para Sasakian manifold was investigated by Jaiswal, Ojha, and Prasad (2002).

The CR-submanifold of a nearly and tightly Lorentzian para-cosymplectic manifold and the CR-submanifold of a nearly and closely hyperbolic cosymplectic manifold are the subjects of the current investigation. Additionally, we have examined invariant and non-invariant hypersurfaces of a Lorentziariparacontact manifold, discussing several intriguing characteristics of a Lorentzian para Sasakian manifold. Additionally, we have examined semi-invariant submanifolds of a roughly Lorentzian para-Sasakian manifold. Additionally, generalized CR-submanifolds of a trans Lorentzian para Sasakian manifold and generalized CR-submanifolds of a trans hyperbolic Sasakian manifold are studied in this work.

2. Manifold Structures:

2.1. Almost complex manifold

A differentiable manifold V_n is referred to as an almost complex manifold and the structure F is referred to as an almost complex structure (Ehresmann, 1947). if there exists a vector valued linear function P of differentiability class C^∞satisfying

$$F^2 = -I, \qquad \text{-----(1)}$$

The vector-valued skew symmetric bilinear function N, which is the Nijenhuis tensor with regard to F, is given by

$$N(X, Y) = [FX, FY] + F[X,Y] - F[FX,Y] - F^2[X,FY] \qquad \text{-----(2)}$$

A complex manifold is an almost complex manifold with vanishing Nijenhuis tensor for all vector fields X,Y of V_n.

2.2 Almost Hermitian manifold:

An almost Hermitian manifold IS an almost complex manifold endowed with a Hermite metric g,

$$g(FX,FY)= g(X, Y),\ g(FX,Y) = -g(X,FY). \qquad \text{-----(3)}$$

The structure {F,g} is called an almost Hermitian structure (Yano, 1965).

Almost Product manifold: If in the differentiable manifold Vn, there exist a vector valued linear function F, of differentiability class C^∞ satisfying

$$F^2 = I, \qquad \text{-----(4)}$$

Then, V n is said to be an almost product manifold (Yano, 1965;Mishra, 1970).

If in an almost product manifold, there exists a Riemannian metric g such that

$$g(F_X,F_Y)=g(X, Y), \qquad \text{-----(5)}$$

and

$$g(F_X, Y)=g(X,F_Y), \qquad \text{-----(6)}$$

Consequently, V_n is referred to as a nearly product Riemannian manifold. If V is the Riemannian connexion in an almost product Riemannian manifold (xF)Y=O, then V n is referred to as an almost decomposable manifold.

2.3 Riemannian manifolds

Let M display the properties of a seamless manifold. A study of geometric qualities in a curved space is made possible by a mathematical construct called a Riemannianmetric, which offers a means of measuring angles and distances on a manifold represented as g on M is a mapp9p, where 9p is a positive definite inner product on TpM. A combination (Mg), where g is a Riemannian metric specified on M, makes up a Riemannian manifold. If a connection V satisfies certain requirements, it is considered a Riemannian connection.

[X,Y]=VxY-VyX -----(7)

And

'0 =6X -----(8)

where any vector fields on M are represented by X and Y.

2.4 Semi Riemannian manifolds

A semi-Riemannian metric g defined on a differentiable manifold M is a continuous mapping on M that yields a non-degenerate symmetric, bilinear structure 9p on T(Tp M) at any point p e M.

(i) 9p(X,Y) = 9p(Y,X), for allX,YeT(TM), -----(9)

(ii) 9p(aX+bY,Z)=agp(X,Z)+bgp(Y,Z),for allX,Y,ZeT(TM) and a,beR -----(10)

(iii) 9p(X,Y) = 0, for allYeT(TM) impliesX = 0, for allX,Ye(TM), -----(11)

2.5 Almost Product manifold

A differentiable manifold V_n is said to be an almost product manifold if there is a vector-valued linear function F of differentiability class C^∞ that satisfies (Yano, 1965; Mishra, 1970).

F^2=I, -----(12)

If a Riemannian metric g exists in a nearly product manifold such that,

g(F_X,F_Y)=g(X, Y), -----(13)

and

g(FX, Y)=g(X,F_Y), -----(14)

then V_n is called an almost product Riemannian manifold. If in an almost product Riemannian manifold (VxF)Y=O, where V is the Riemannian connexion, then V n is called an almost decomposable manifold.

2.6 Almost contact manifold:

If in an odd dimensional differentiable manifold V n there exist a vector valued linear function F, a vector field U and a l-form u, satisfying

2. F = -I+u ® U FU=O, -----(15)

then $V_n$is called an almost contact manifold and {F,U,u} is called an almost contact structure (Sasaki and Hatakeyama, 1961). In an almost contact manifold

' u(U) = 1, uo F=O. -----(16)

On a non-invariant hypersurface of an almost Hermitian manifold, an almost contact manifold can be formed. An essentially contact manifold where the metric tensor g satisfies

$$g(F_X,F_Y)=g(X, Y)-u(X)u(Y), \qquad \text{-----(17)}$$

is called an almost contact metric manifold and {F,U,u,g} is called an almost contact metric structure.

An almost contact metric manifold for which

$$VxF = 0, \qquad \text{-----(18)}$$

is called a Cosymplectic manifold (Goldberg, 1968). An almost contact metric manifold for which

$$(VxF)X=O, \qquad \text{-----(19)}$$

is called a nearly Cosymplectic manifold (Blair, 1971). An almost contact metric manifold is called a closely Co symplectic if F is a Killing and u is a closed, that is (Blair and Shower, 1974)

$$VuF=O, VU=O, Vu=O. \qquad \text{-----(20)}$$

An almost contact metric structure (F,U,u,g) is called a Sasakian manifold if (Blair, 1990)

$$(VxF)(Y)=g(X, Y)U-u(Y)X, \qquad \text{-----(21)}$$

and trans Sasakian if (Blair and Oubina, 1990)

$$(VxF)(Y)=u(g(X, Y)\, U - u(Y)X)+\sim(g(F_X, Y)\, U - u(Y)F_X), \qquad \text{-----(22)}$$

for function u, ~. An almost contact metric manifold IS called a nearly Sasakian manifold if (Blair, 1976)

$$(VxF)(Y)+ (VyF)(X)=u(Y)X+u(X)Y-2g(X,Y)U. \qquad \text{-----(23)}$$

3. Symmetric LP-Sasakian manifold with a type of Semi-Symmetric Connection

3.1 Generalized Pseudo Ricci symmetric LP-Sasakian Manifold with a type of Semi-Symmetric Connection

A non-flat differentiable manifold (Mn; g) (n > 3) is called generalized pseudo symmetric G(PS)n, if there exists a vector field L and 1-form A, B, C, D on Mn such that

$$(\nabla_X S)\,(Y, Z) = 2A(X)S\,(Y, Z) + B(R(X, Y, Z)) + C\,(Y)S\,(X, Z)+ D(Z)S(X, Y) + L(R\,(X, Z, Y)); \qquad \text{-----(24)}$$

where S, R are Ricci tensor and Curvature tenso with respect to r, theoperator of covariant differentiation with respect to g.Chaki and Koley introduced another type of non-flat differentiable manifold Mn; g) (n > 2) in 1993, satisfies the condition

$$(\nabla_X S)\,(Y, Z) = 2A(X)S\,(Y, Z) + B(Y)S\,(X, Z) + C(Z)S(X, Y); \qquad \text{-----(25)}$$

3.2 Semi-symmetric non-metric connection E in a LP-Sasakian manifold

Let Mn be an n-dimensional LP-Sasakian manifold equipped with the Levi-Civita connection r of its Lorentzian metric g. S. Kumar and J. Upreti [48]define a new connection B in Mn by

$$\text{(a) } B_X Y = \nabla_X Y + \eta\,(Y)X - g(X, Y)\xi + F(X, Y)\xi,) \qquad \text{-----(26)}$$

$$\text{(b) } B_X g = 2\eta(X)g, \qquad \text{-----(27)}$$

$$\text{(c) } g(\Phi X, Y) - g(\Phi Y,X) = 0: \qquad \text{-----(28)}$$

In view of equation (26), the torsion tensor T of Mn with respect to connectionB is given by

$$T(X, Y) = \eta(Y)X - \eta(X)Y, \qquad \text{-----(29)}$$

Further for a 1-form η on M_n, we have

$$B_X(\eta\,(Y)) = (BX\,\eta)Y + \nabla X(\eta\,(Y)) - (\nabla X\,\eta)Y+ \eta\,(X)\,\eta\,(Y) + g(X, Y) - g(\Phi X, Y) \qquad \text{-----(30)}$$

which implies

$(B_X \eta)Y = (\nabla X \eta)Y - g(\Phi X, \Phi Y) + g(\Phi X, Y),$ -----(31)

using tensor field in the above expression, we have

$(B_X \eta)Y = 2g(\Phi X; Y) -g(\Phi X; \Phi Y),$ -----(32)

Covariantdifferentiation of the torsion tensor T is given by

$B_X(T(Y,Z)) = (B_X T)(Y,Z) + T(B_X Y, Z) + T(Y, B_X Z),$ -----(33)

using equation 29 in above equation, we get

$(B_X T)(Y,Z) = ((B_X \eta)Z)Y - ((BX \eta) Y)Z.$ -----(34)

Now let us define

$\tilde{T}(X, Y, Z) = g(T(X, Y), Z),$ -----(35)

In view of equation 29, we have

$\tilde{T}(X, Y, Z) + \tilde{T}(Y, Z, X) + \tilde{T}(Z, X, Y) = 0.$ -----(36)

In a LP Sasakian manifold, we have

$B_X\xi = \nabla_X\xi - \Phi^2 X,$ -----(37)

using $\nabla_X\xi = \varphi X$ in the above equation, we have

$B_X\xi = \Phi X - \Phi^2 X$ -----(38)

3.3 Curvature tensor of M_n with respect to connection B

In a LP-Sasakian manifold the curvature tensor $\tilde{R}$ (X, Y, Z) of a LP-Sasakianmanifold M_n with respect to the connection B, defined as [48]

$\tilde{R}(X, Y, Z) = R(X, Y, Z) + 2g(\varphi X, Z)Y - 2g(\varphi Y, Z)X - g(Y, Z)(\varphi X)X, Z)(\varphi Y) + \varphi Y, Z)(\varphi X) - \varphi X, Z)(\varphi Y)Y$
$X - \eta(X)\eta(Z)Y + g(Y,Z)X - g(X,Z)Y - \eta(X)g(\varphi Y,Z)\xi + \eta(Y)g(\varphi X, Z)\xi,$ -----(39)

where $R(X, Y, Z) = \nabla_X \nabla_Y \nabla_Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]}Z,$ is curvature tensorof $M_n$with respect to the Riemannian connection ∇. Let K and K be thecurvature tensors of type (0, 4) given by

$K(X, Y, Z, U) = g(R(X, Y, Z), U),$ -----(40)

and

$\tilde{K}(X, Y, Z, U) = g(R(X, Y, Z), U).$ -----(41)

Theorem 3.1. *In an LP-Sasakian manifold with connection B, we have*

$R(X, Y, Z) + R(Y, Z, X) + R(Z, X, Y) = 0$ -----(42)

$K(X, Y, Z, U) + K(Y, X, Z, U) = 0$ -----(43)

Proof.Using (59) and the first Bianchi identity

$R(X, Y, Z) + R(Y, Z, X) + R(Z, X, Y) = 0$ -----(44)

with respect to Levi-Civita connection ∇, we get (62). From (59), we have

$K(X,Y,Z,U) = K(X,Y,Z)U + 2g(\varphi X,Z)g(Y,U)\varphi Y,ZX,U) - Y,Z\varphi XU)XZ\varphi Y,U) + \varphi Y,Z)g(\varphi X,U) - g(\varphi X,Z)g(\varphi Y,U) + \eta(Y)\eta(Z)g(X,U) - \eta(X)\eta(Z)g(Y,U) + g(Y,Z)g(X,U) - g(X,Z)g(Y,U) - \eta(X)g(\varphi Y,Z)\eta(U) + \eta(Y)g(\varphi X, Z)\eta(U)$ -----(45)

Since, $K(X, Y, Z, U) = -K(Y, X, Z, U)$, we get equation (45)

Let *Mn*be an *n*-dimensional LP-Sasakian manifold. Then the Ricci ten sor$\tilde{S}$of the manifold *Mn*with respect to the connection *B* is defined by

$$\tilde{S}(X, Y) = \sum_{t=1}^{n} \varepsilon 1 g(\tilde{R}(ei, X, Y), ei), \quad \text{-----(46)}$$

and the scalar curvature of the manifold *Mn*with respect to the connection*E*is given by

$$\tilde{r} = \sum_{t=1}^{n} \varepsilon 1 \tilde{R}(ei, ei), \quad \text{-----(47)}$$

where *{e*1*, e*2*, ..., en}* is an orthonormal frame and *εi= g(ei,ei)*.

Theorem 3.2. *In an LP-Sasakian manifold the Ricci tensor S and scalarcurvature r of connection B are given by*

$$\tilde{S}(Y, Z) = \bar{S}(Y, Z) - (2n-4)g(\varphi Y, Z) + (n-2)g(\varphi Y, \varphi Z) \quad \text{-----(48)}$$

3.4. Curvature Tensor of Mnwith Respect to Connection B

$$\tilde{r} = r - (3n - n2 - 2), \quad \text{-----(49)}$$

where $\bar{S}$*and r denote the Ricci tensor and scalar curvature of Levi-Civitaconnection,∇ respectively. Consequently,* $\tilde{S}$*is symmetric.*

Proof.In view of (52) and (58), we have

$$\tilde{S}(Y,Z) = \sum_{t=1}^{n} \varepsilon 1 (R(ei, Y, Z), ei) - (2n-4)g(\varphi Y, Z) + (n-2)g(\varphi Y, \varphi Z) \quad \text{----(50)}$$

Since the Ricci tensor of Levi-Civita connection ∇is given by

$$\bar{S}(Y, Z) = \sum_{t=1}^{n} \varepsilon 1 g(R(ei, Y, Z), ei), \quad \text{----(51)}$$

then (69) implies(67). (68) follows from (67). Also from (67), it is obvious that *S* is symmetric.

Lemma- Let *Mn*be a *n*-dimensional LP-Sasakian manifold with thesemi- symmetric non-metric connection *B*. Then

(a) $\tilde{R}(\xi, X, Y) = 2g(X, Y)\xi - g(\varphi X, Y)\xi + \eta(Y)(\varphi X) + \eta(X)\eta(Y)\xi - \eta(Y)X,$ ----(52)

(b) $\tilde{R}(\xi, X, \xi) = \varphi^2 X - \varphi X,$ ----(53)

(c) $\tilde{R}(X, Y, \xi) = \eta(Y)\{X - (\varphi X)\} - \eta(X)\{Y - (\varphi Y)\},$ ----(54)

(d) $\tilde{R}(\xi, \xi, \xi) = 0$ ----(55),

(e) $\tilde{S}(X, \xi) = S(X, \xi) = (n-1)\eta(X),$ ----(56)

(f) $\tilde{S}(\varphi X, \varphi Y) = S(X, Y) + (n-1)\eta(X)\eta(Y)2(n-2)X, \varphi Y.$ ----(57)

Proof.using (41)(43)(44),$\nabla_X \xi = \varphi X$, in (67), we have all the above results

3.5 Generalized pseudo Ricci symmetric LPSasakianmanifold admitting a semi-symmetricnon-metric connection B

Anon-flat Riemannian manifold *Mn*is called Generalized pseudo Ricci symmetric LP-Sasakian manifold with respect to a semi-symmetric non metricconnection *B* if

$$(B_X\tilde{S})(Y,Z) = 2A(X)\tilde{S}(Y,Z) + B(Y)S(X,Z) + C(Z)\tilde{S}(X,Y) \quad \text{----(58)}$$

for all vector fields *X, Y, Z.* Putting $Z = \xi$ in above equation, we get

$$(B_X\tilde{S})(Y,\xi) = \lambda(X)\tilde{S}(Y,\xi) + \mu(Y)\tilde{S}(X,\xi) + \nu(\xi)\tilde{S}(X, Y), \quad \text{----(59)}$$

By the virtue of (58) the above equation gives

$$2\bar{A}(X)\tilde{S}(Y,\xi) + \bar{B}(Y)\tilde{S}(X,\xi) + \bar{C}(\xi)\tilde{S}(X,Y) \quad \text{----(60)}$$

$$= (n-1)\{(B_X\eta)Y\} - \tilde{S}(Y, \bar{X} - \bar{\bar{X}}) \quad \text{----(61)}$$

3.6 Generalized Pseudo Ricci Symmetric LP-Sasakian Manifold Admitting a Semi Symmetric Non-Metric Connection B

Using (52)(e) in above equation, we have

$$2\bar{A}(X)(n-1)\eta(Y) + \bar{B}(Y)(n-1)\eta(X) + \bar{C}(\xi)\tilde{S}(X, Y) = (n-1)(B_X\eta)\, Y\} - \tilde{S}(Y, \bar{X} - \bar{\bar{X}}) \quad \text{----(62)}$$

Setting $X = Y = \xi$ in above equation, we obtain

$$(n-1)\{2\bar{A}(\xi) + \bar{B}(\xi) + \bar{C}(\xi)\} = 0 \quad \text{----(63)}$$

which implies that

$$2\bar{A}(\xi) + \bar{B}(\xi) + \bar{C}(\xi) = 0 \quad \text{----(64)}$$

Putting $X = \xi$ in (62), we get

$$2\bar{A}(\xi)(n-1)\eta(Y) - \bar{B}(Y)(n-1) + \bar{C}(\xi)(n-1)\eta(Y) = 0. \quad \text{----(65)}$$

or

$$B(Y) = -\eta(Y)\{\bar{B}(\xi)\} \quad \text{----(66)}$$

Putting $Y = \xi$ in (62), we get

$$-2\bar{A}(X)(n-1) + \bar{B}(\xi)(n-1)\eta(X) + \bar{C}(\xi)(n-1)\eta(X) = 0, \quad \text{----(67)}$$

$$-2\bar{A}(X) + \bar{B}(\xi)\eta(X) + \bar{C}(\xi)\eta(X) = 0 \quad \text{----(68)}$$

Using (64) in above equation, we have

$$\bar{A}(X) = -\bar{A}(\xi)\eta(X). \quad \text{----(69)}$$

Since $(\nabla_\xi\tilde{R})(\xi, X) = 0$, then from (59) it can be shown that

$$2\bar{A}(\xi)\tilde{S}(\xi, X) + \bar{B}(\xi)\tilde{S}(\xi, X) + \bar{C}(X)\tilde{S}(\xi, \xi) = 0, \quad \text{----(70)}$$

$$(n-1)\eta(X)\{2\bar{A}(\xi)X + \bar{B}(\xi)\} - \bar{C}(X)(n-1) = 0, \quad \text{----(71)}$$

Using (64) in the above equation, we have

$$\bar{C}(X) = -\bar{C}(\xi)\eta(X). \quad \text{----(72)}$$

Adding (66), (68) and (73), we get

$$2\bar{A}(X) + \bar{B}(X) + \bar{C}(X) = -\{2\bar{A}(\xi) + \bar{B}(\xi) + \bar{C}(\xi)\}\eta(X), \quad \text{----(73)}$$

Using (64), we have

$$2\bar{A}(X) + \bar{B}(X) + \bar{C}(X) = 0, \quad \text{----(74)}$$

then

3.7 Generalized Ricci-Recurrent Lp-Sasakian Manifolds with Connection B Admitting Co-Dazzi Type Ricci Tensor

$$2\bar{A} + \bar{B} + \bar{C} = 0, \quad \text{----(75)}$$

for any vector field X on Mn.

We know that

from(58) we have

$$\bar{A}(X)\tilde{S}(Y, Z) + \bar{B}(X)g(Y, Z) = B_X\tilde{S}(Y, Z) - \tilde{S}(B_XY, Z) - \tilde{S}(Y, B_XZ) \quad \text{----(76)}$$

putting $Z = \xi$ and using (52) (f) we get

$$(n-1)\bar{A}(X)\eta(Y) + \bar{B}(X)\eta(Y) = B_X\tilde{S}(Y, \xi) - \tilde{S}(B_XY, \xi) - \tilde{S}(Y, B_X\xi) \quad \text{----(77)}$$

Putting the values from(60) in (73) we get

$$(n-1)\bar{A}(X)\eta(Y) + B(X)\eta(Y) = (n-1)[(B_X\eta)Y] - \tilde{S}(Y, X - X \quad \text{----(78)}$$

putting $Y = \xi$ we get

$$(n-1)\bar{A}(X) + \bar{B}(X) = 0 \qquad \text{----(79)}$$

Here we assumed that the generalised Ricci-recurrent manifold n withconnection B admits Co-dazzi type Ricci tensor, then we have

$$(B_X\tilde{S})(Y, Z) = (B_Y\tilde{S})(X, Z). \qquad \text{----(80)}$$

Using (58) in above equation, we get

$$\bar{A}(X)\tilde{S}(Y, Z) + \bar{B}(X)g(Y, Z) = \bar{A}(Y)\tilde{S}(X, Z) + \bar{B}(Y)g(X, Z). \qquad \text{----(81)}$$

Putting $X = \xi$ in above equation, we have

$$\bar{A}(\xi)\tilde{S}(Y, Z) + \bar{B}(\xi)g(Y, Z) = \{(n-1)\bar{A}(Y) + \bar{B}(Y)\}\eta(Z).$$

Using (79), in above equation, we get

$$\bar{A}(\xi)\tilde{S}(Y, Z) + \bar{B}(\xi)\, g(Y, Z) = 0, \qquad \text{----(82)}$$

$$\tilde{S}(X, Y) = \mu\, g(X, Y). \qquad \text{----(83)}$$

Theorem 3.3. *If a generalised Ricci-recurrent LP-Sasakian manifold withconnection Badmits a Cyclic Ricci tensor then it becomes an Einstein manifold) with constant*

$$\mu = -\frac{\bar{B}(\xi)}{\bar{A}(\xi)} \qquad \text{----(84)}$$

3.8 Generalized Ricci-recurrent LP-Sasakian, manifolds with connection B admitting cyclic type Ricci tensor

In this section we suppose that a generalized Ricci-recurrent LP-Sasakianmanifolds *Mn*with connection B admits a cyclic Ricci tensor S, i.e.

$$(B_X\tilde{S})(Y, Z) + (B_Y\tilde{S})(Z, X) + (B_Z\tilde{S})(X, Y) = 0 \qquad \text{----(85)}$$

Then by virtue of (26)(a) above equation can be written as

$$\bar{A}(X)\tilde{S}(Y, Z) + \bar{B}(X)g(Y, Z) + \bar{A}(Y)\tilde{S}(Z, X) + \bar{B}(Y)g(Z, X) + \bar{A}(Z)\tilde{S}(X, Y) + \bar{B}(Z)g(X, Y) = 0 \qquad \text{----(86)}$$

Putting $Z = \xi$ in above equation

$$\bar{A}(X)\tilde{S}(Y, \xi) + \bar{B}(X)\eta(Y) + \bar{A}(Y)\tilde{S}(\xi, X) + \bar{B}(Y)\eta(X) + \bar{A}(\xi)\tilde{S}(X, Y) + \bar{B}(\xi)g(X, Y) = 0 \qquad \text{----(87)}$$

from (52)(e) we have

$$(n-1)\bar{A}(X)\eta(Y) + \bar{B}(X)\eta(Y) + (n-1)\eta(X)\bar{A}(Y) + \eta(X)\bar{B}(Y)X, Y) + X, Y) = 0 \qquad \text{----(88)}$$

or

$$\eta(Y)\{(n-1)\bar{A}(X) + \bar{B}(X)\} + \eta(X)\{(n-1)\bar{A}(Y) + \bar{B}(Y)\} + \bar{A}(\xi)\tilde{S}(X, Y) + \bar{B}(\xi)g(X, Y) = 0, \qquad \text{----(89)}$$

if the manifold contains co-dazzi type ricci tensor then

$$(n-1)\bar{A}(X) + \bar{B}(X) = 0 \qquad \text{----(90)}$$

from (62) and (79) we have

$$\bar{A}(\xi)\tilde{S}(X, Y) = -\bar{B}(\xi)g(X, Y), \qquad \text{----(91)}$$

$$\tilde{S}(X, Y) = -\frac{\bar{B}(\xi)}{\bar{A}(\xi)}g(X, Y) \qquad \text{----(92)}$$

$$\tilde{S}(X, Y) = v\, g(X, Y). \qquad \text{----(93)}$$

Where

$v = -\frac{\bar{B}(\xi)}{\bar{A}(\xi)}$ ----(94)

Theorem 3.4. *If a generalised Ricci-reccurent LP-Sasakian manifold with*connection B admits a cyclic Ricci tensor, then it became an Einstein manifold.

4. **Semi Pseudo Symmetric LP-Sasakian Manifold (Sps)N (Mn;G) Admitting A Semi-symmetric Non Metric Connection B**

Semi pseudo symmetric LP-Sasakian manifold(SPS)n (Mn; g) admitting a semi-symmetric non metric connection B

A non-flat Riemannian manifold *n* is called Semi pseudo symmetric LP-Sasakian manifold with respect to a semi-symmetric non metric connection*B* if

$(B_x \tilde{R})(Y, Z, W) = 2\bar{A}(X)\tilde{R}(Y, Z, W) + \bar{A}(Y)\tilde{R}(X, Z, W) + \bar{A}(Z)\tilde{R}(Y, X, W) + \bar{A}(W)\tilde{R}(Y, Z, X)$ ----(95)

Contracting above equation with respect to *W* and after that putting $Z = \xi$,we get

$(B_X\tilde{S})(Y, \xi) = 2\bar{A}(X)(n-1)\eta(Y) + \bar{A}(Y)(n-1)\eta(X) + \bar{A}(\xi)\tilde{S}(Y, X) + \eta(Y)\bar{A}(X - X) - \eta(X)\bar{A}(Y - Y)$

----(96)

where *A* is a non-zero 1-form and

$g(X, P) = \bar{A}(X) \Rightarrow \bar{A}(\xi) = g(\xi, P) = \eta(P).$ ----(97)

Using (59) in (96), we get

$(n-1)\{(B_X\eta)\ Y\} - \tilde{S}(Y, \bar{X} - \bar{\bar{X}}) = 2\bar{A}(X)(n-1)\eta(Y) + \bar{A}(Y)(n-1)\eta(X) + \eta(P)\tilde{S}(Y, X) + \eta(Y)\bar{A}(X - X) - \eta(X)\bar{A}(Y - Y)$ ----(98)

Putting X = ξ in above equation, we get

$(3n-2)\ (P)\eta(Y) - (n-1)\ \bar{A}\ (Y) + Y - \bar{Y}) = 0.$ ----(99)

Putting Y = ξ in above equation, we get

$\eta(P) = 0.$ ----(100)

Putting the value of η(P) in (100), we get

$\bar{A}\ (Y) = 0.$ ----(101)

5. **On a Type of Semi Symmetric Non- Metric s-Connection on Hsu- Kähler Manifold**

Semi-symmetric non-metric s-connection

The afine metric connection *B* satisfying

$(B_X\Phi)(Y) = \eta\ (Y)X - g\ (X, Y)\xi,$ ----(102)

is called s-connection. Where ξ is 1-form and η is a vector field associatedwith ξ.

A s-connection *B* is called semi-symmetric non-metric s-connection if

$B_XY = \nabla_XY - \eta(Y)X - g(X, Y)\xi.$ ----(103)

Also

$(B_Xg)(Y, Z) = 2\eta(Y)g(X, Z) + 2\eta(Z)g(X, Y).$ --------(104)

The torsion tensor *S* of *Mn*with respect to the connection *B*is given by

$S(X, Y) = \eta(X)Y - \eta\ (Y)X,$ --------(105)

where

$S(X, Y) = B_X Y - B_Y X - [X, Y].$ --------(106)

$g(X, \xi) = \eta(X).$ --------(107)

Let us define

$P(X, Y, Z) = -\eta(Y)g(X, Z) - \eta(Z)g(X, Y),$ --------(108)

Now, we have

$B_X(\eta(Y)) = (B_X\eta)Y + \eta(B_X Y)$ --------(109)

$= (B_X\eta)Y + \nabla_X(\eta(Y)) - (\nabla_X\eta)Y - \eta(X)\eta(Y) - g(X, Y)\eta(\xi),$ --------(110)

which implies

$(B_X\eta)Y = (\nabla_X\eta)Y + \eta(X)\eta(Y) + g(X, Y)\eta(\xi).$ --------(111)

$\nabla_X\xi = X,$ --------(112)

6. **Curvature tensor of Hsu-unified structuremanifold equipped with the semi-symmetric on-metric s-connection** B

According to definition of curvature tensor of a unified structure manifold$\bar{M}n$with respect to the Levi -civitaconnection, we define the curvaturetensor of n with respect to new connection, by

$\bar{R}(X, Y, Z) =$ BxByBz- ByBx Z $- B_{[X,Y]}Z.$ --------(113)

In a Hsu-unified structure manifold equipped with a semi-symmetric s-connection$E$, the curvature tensor $\bar{R}(X, Y, Z)$ is given by

$\bar{R}(X, Y, Z) = R(X, Y, Z) - \beta(X, Z)Y + \beta(Y, Z)X - g(Y, Z)LX + g(X, Z)LY$ --------(114)

where $\bar{R}(X, Y, Z)$ is curvature tensor of $Mn$with respect to the Riemannianconnection ∇, β is a tensor field of type (0, 2) defined by

$\beta(X, Y) = (\nabla X\eta)Y + \eta(X)\eta(Y) + g(X, Y)\eta(\xi),$ --------(115)

and

$LX = (\nabla_X\xi - \eta(X)\xi)$ --------(116)

Let K and K be the curvature tensors of type (0, 4) given by

$K(X, Y, Z, U) = g(R(X, Y, Z), U),$ --------(117)

$\bar{K}(X, Y, Z, U) = g(\bar{R}(X, Y, Z), U).$ --------(118)

Conclusion:

The systematic and rigorous study of Lorentzian Para-Sasakian (LP-Sasakian) manifolds with semi-symmetric non-metric connections is the focus of this work, with particular attention to different generalized curvature requirements. Examining how the intrinsic and extrinsic geometric features of LP-Sasakian manifolds are altered by the addition of a semi-symmetric non-metric connection and characterizing the resultant structures under various curvature limits have been the main goals.The basic ideas of LP-Sasakian manifolds were reviewed and expanded in the presence of a semi-symmetric non-metric connection in the first section of the research. With regard to this link, explicit formulas for the torsion tensor, curvature tensor, Ricci tensor, and scalar curvature were obtained.These formulations clearly show that the underlying connection has a considerable impact on the manifold's curvature properties, producing results that are very different from those obtained under the Levi-Civita connection. A number of significant identities and lemmas were developed, offering a fundamental basis for further research.

Additionally, the notion of generalized pseudo Ricci symmetric LP-Sasakian manifolds permitting a semi-symmetric non-metric connection was presented and thoroughly examined. The crucial conclusion that these forms meet particular linear connections was reached by obtaining necessary and sufficient criteria involving linked 1-forms. Understanding the structural limitations imposed by generalized pseudo Ricci symmetry in the LP-Sasakian scenario is greatly aided by this conclusion.

Lastly, we studied semi-pseudo symmetric and semi-pseudo Ricci symmetric LP-Sasakian manifolds that accept a semi-symmetric non-metric connection. Important non-existence theorems were established by proving that such manifolds cannot exist for dimensions larger than three. These findings demonstrate that when semi-symmetric non-metric connections are present, LP-Sasakian structures and certain curvature symmetry criteria are inherently incompatible.

Overall, by clarifying the interaction between non-Riemannian connections and particular geometric features, the conclusions gained in this study greatly enhance the theory of LP-Sasakian manifolds. In addition to generalizing a number of previously established findings, the conclusions also provide new avenues for investigation into contact geometry, Lorentzian geometry, and their applications in mathematical physics.

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