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GRAPH-THEORETIC CHARACTERIZATION OF SOFT GENERALIZED** Ψ-CLOSED SETS IN SOFT TOPOLOGICAL SPACES VIA BIPARTITE STRUCTURES

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ABSTRACT

The concept of soft sets was introduced by Dmitri Molodtsov as a mathematical tool for handling uncertainty and imprecise information, and has since been widely applied to decision-making problems involving vague or ambiguous data. In this paper, we introduce a new class of soft sets, namely soft generalized** Ψ -closed sets (briefly, $Sg^{**}\psi$ *-closed sets*) and soft generalized** Ψ -open sets (briefly, $Sg^{**}\psi$ *-open sets*), in the framework of soft topological spaces. We examine the relationships between $Sg^{**}\psi$ -closed sets and several existing classes of soft closed sets, and establish some of their fundamental properties. Furthermore, we propose a novel approach for representing soft sets by means of bipartite graphs, and show that various operations on soft sets—such as union, intersection, and logical operations—can be modeled and visualized through bipartite graph structures. And we introduce a bipartite graph representation of $Sg^{**}\psi$ -closed sets and to investigate their associated operations within this graphical framework.

Keywords: $Sg^{**}\psi$-closed set, $Sg^{**}\psi$ -open set, $Sg^{**}\psi$ -closure, $Sg^{**}\psi$ -interior, $Sg^{**}\psi$ -neighborhood, $Sg^{**}\psi$ –closed bipartite graph, $Sg^{**}\psi$ – closed complete bipartite graph.

I - INTRODUCTION

Soft set theory, introduced by D. Molodtsov in 1999[14], provides an effective mathematical framework for addressing uncertainties through parameterized structures. The concept of soft topological spaces was later established by Muhammad Shabir and Munazza Naz in 2011[11], which laid the foundation for studying topological properties within the framework of soft sets. Subsequent developments by Naim Çağman[2] and collaborators further enriched the theory by extending definitions and exploring structural properties of soft sets. In classical topology, Generalized closed sets were introduced by Norman Levine in 1970[9], which motivated many researchers to investigate various generalized forms of closed and open sets. Building upon this concept, V. Kannan defined soft generalized closed and soft generalized open sets in soft topological spaces[8] . Earlier studies by M. K. R. S. Veerakumar[24] examined sets lying between closed sets and generalized closed sets. Later, the notion of g∗-closed sets in topological spaces was proposed by M. Pauline Mary Helen, Veronica Vijayan, and Ponnuthai Selvarani[16]. Further extensions were introduced in the context of soft topology, where A. Devika[3] defined soft g∗-closed sets. Subsequently, A. Kalavathi and G. Sai Sundara Krishnan[7] introduced both soft g∗-closed and soft g∗-open sets in soft topological spaces. In addition, M. A. Abd Allah and A. S. Nawar[1] investigated ψ^*-closed sets in fuzzy topological spaces, while more recently N. Gomathi and T. Indira[5] proposed soft g∗∗-closed sets. The fundamental operations of soft sets, including union, intersection, AND-operation, and OR-operation, were formulated by P. K. Maji[10] and co-authors, which significantly strengthened the theoretical framework of soft set theory. Parallel to these developments, graph theory has evolved as an essential tool for modeling relationships and structures. In 2014[23], Rajesh K. Thumbakara and Boben George proposed the concept of soft graphs, extending soft set theory to graph structures. This idea was further developed by Sumit Mohinta and T. K. Samanta[13] through the introduction of fuzzy soft graphs, which combine the characteristics of fuzzy sets and soft sets in graph models. Additionally, Faiz farid[4] and co-researchers proposed new approaches for representing graphs using adjacency relations within the soft set framework. Motivated by this research gap, the present work introduces $Sg^{**}\psi$ -closed sets and $Sg^{**}\psi$ -open **sets** in soft topological spaces. Furthermore, this study proposes a novel method for representing bipartite graphs using $Sg^{**}\psi$ -closed sets. The results obtained in this work contribute to strengthening the connection between soft topology and graph theory, and they may provide new perspectives for modeling relationships in uncertain environments.

II-PRELIMINARIES

This section introduces essential definitions that are useful for understanding the ideas discussed in this paper and for developing the results presented in this study.

Definition : 2.1

A soft set F_A on the universe U is defined by the set of ordered pairs, E be the set of parameters and AE, then $F_A = \{(x, f_A(x)) : xE\}$ where $f_A :\ E\,P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$. Here the value of $f_A(x)$ may be aribitary. Some of them may be empty some may have non-empty intersection.

Note that the set of all soft sets with the parameter set E over U will be denoted by $S(U)$.

Definition : 2.2

Let $F_A S(U)$. If $f_A(x) = \emptyset$ for all $x \in A$ then F_A is called an empty soft set, denoted by $F_\emptyset$.

Definition: 2.3

Let $F_A S(U)$. If $F_A(x) = U$ for all xA thenF_A is called a A-universal soft set, denoted by $F_{\tilde{A}}$. If $A = E$, then the A-universal soft set is called universal soft set denoted by $F_{\tilde{E}}$.

Definition: 2.4

Let $F_A, F_B S(U)$. Then soft union$F_A \tilde{\cup} F_B$, Soft intersection $F_A \tilde{\cap} F_B$, and soft difference $F_A \setminus F_B$ of F_A and F_A are defined by respectively. $f_{A\tilde{\cup}B}(x) = f_A(x) \cup f_B(x)$, $f_{A\tilde{\cap}B}(x) = f_A(x) \cap f_B(x)$, $f_{A\tilde{\setminus}B}(x) = f_A(x) \setminus f_B(x)$,and the soft complement $\tilde{F^{c}}_{A}$ of F_A is defined by $\tilde{f^{c}}_{A}(x) = f^{c}_{A}$ where $f^{c}_{A}(x)$ is complement of the set $f_A(x)$, that is $f^{c}_{A}(x) = U\setminus f_A(x)$ for all xE.

Definition: 2.5

Let $F_A \in S(U)$. The relative complement of F_A is denoted by F'_A and is defined by $\left(F_A\right)' = F_A'$) where $F'_A: A\rightarrow P(U)$ is a mapping given by $F'_{\propto} = U\setminus F_{\propto}$ for all $\propto\ \in A$.

Definition: 2.6

Let $F_A \in S(U)$. A soft topology on F_A denoted by $\tilde{\tau}$, is a collection of soft subsets of F_A having following conditions .

(i) $F_A, F \ \in \tilde{\tau}$.

(ii) The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$

(iii) The intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$

Then the pair $(F_A, \tilde{\tau})$ is called a soft topological space.

Definition: 2.7

Let $(F_A, \tilde{\tau})$ be a soft topological space, then every element of $\tilde{\tau}$ is called a soft open sets in $\tilde{\tau}$.

Definition: 2.8

Let $(F_A, \tilde{\tau})$ be a soft topological space. A soft set F_A is said to be a soft closed set, if its relative complement F'_A belongs to $\tilde{\tau}$.

Definition: 2.9

Let $(F_A, \tilde{\tau})$ be a soft topological space, then soft interior of soft set F_A is defined as the union of all soft open sets contained in F_A. It is denoted by $int(F_A)$.

Definition : 2.10

Let $(F_A, \tilde{\tau})$ be a soft topological space, then soft closure of soft set F_A is defined as the intersection of all soft closed super sets containing in F_A. It is denoted by $cl(F_A)$.

Definition : 2.11

Let $(F_A, \tilde{\tau})$ be a soft topological space, a soft set F_A over U is called a

1) Soft semi open set if $F_A\ cl\left(int\left(F_A\right)\right)$ and a semi closed set if $int\left(cl\left(F_A\right)\right)F_A$.
2) Soft pre open set if $F_A\ int\left(cl\left(F_A\right)\right)$ and a pre closed set if $cl\left(int\left(F_A\right)\right)F_A$.
3) Soft α-open set if $F_A int(cl(int(F_A)))$ and a α-closed set if $(int(cl(F_A)))\ F_A$.
4) Soft β-open set if $F_A\ cl(int(cl(F_A)))$ and a β-closed set if $int(cl(int(F_A)))\ F_A$

5) Soft regular open set if $F_A = int(cl(F_A))$ and a regular closed set if $F_A = cl(int(F_A))$

6) Soft generalized closed (briefly soft g-closed) set if $cl(F_A)U_A$, whenever $F_A U_A$ and U_A is soft open in $(F_A, \tilde{\tau})$.

7) Soft semi generalized closed (briefly soft sg-closed) set if $scl(F_A)\ U_A$, whenever $F_A U_A$ and U_A is soft semi open in $(F_A, \tilde{\tau})$

8) Soft generalized * closed (briefly soft g*-closed) set if $cl(F_A)U_A$, whenever $F_A U_A$ and U_A is soft g-open in $(F_A, \tilde{\tau})$.

9) Soft α-generalized closed (briefly α soft g-closed) set if α $cl(F_A)U_A$, whenever $F_A U_A$ and U_A is soft open in $(F_A, \tilde{\tau})$.

10) Soft generalized**-closed (briefly soft g**-closed) set if $cl(F_A)\ U_A$, whenever $F_A U_A$ and U_A is soft g*-open in $(F_A, \tilde{\tau})$.

11) Soft ψ -closed (briefly soft ψ -closed) set if $scl(F_A)\ U_A$, whenever $F_A U_A$ and U_A is soft generalized open in $(F_A, \tilde{\tau})$.

12) Soft α ψ -closed (briefly soft α ψ -closed) set $\psi cl(A) \subseteq U$ if, whenever $F_A U_A$ and U_A is soft α open in $(F_A, \tilde{\tau})$.

13) Soft ψ g-closed (briefly soft ψ g-closed) set $\psi cl(A) \subseteq U$ if, whenever $F_A U_A$ and U_A is soft open in $(F_A, \tilde{\tau})$.

14) Soft ψ *α-closed (briefly soft ψ *α -closed) set $\alpha cl(A) \subseteq U$ if, whenever $F_A U_A$ and U_A is soft ψ g open in $(F_A, \tilde{\tau})$.

15) Soft g*ψ -closed (briefly soft g*ψ -closed) set $\psi cl(A) \subseteq U$ if, whenever $F_A U_A$ and U_A is soft g- open in $(F_A, \tilde{\tau})$.

16) Soft ψ *g*-closed (briefly soft ψ *g*-closed) set $\psi cl(A) \subseteq U$ if, whenever $F_A U_A$ and U_A is soft ψ g- open in $(F_A, \tilde{\tau})$.

The relative complements of the above soft closed sets are called Soft open sets respectively.

III – SOFT GENERALIZED**Ψ- CLOSED SETS IN SOFT TOPOLOGICAL SPACES

In this section, we introduce the notion of soft generalized **Ψ-closed sets and examine their relationships with the existing classes of soft closed sets in a more comprehensive manner.

Definition:3.1

A soft set F_A is called a soft generalized **Ψ- closed (briefly $Sg^{**}\psi$ - closed) set if $\psi cl(F_A)U_A$, whenever $F_A U_A$ and U_A is soft g*- open in $(F_A, \tilde{\tau})$.

Example :3.2

Let U=$\{\alpha, \beta, \gamma\}$, $E = \{J_1, J_2, J_3\}$ and $A = \{J_1, J_2\}E$, $F_A = \{(J_1, \{\alpha, \beta\}), (J_2, \{\beta, \gamma\})\}$ $F_{A_1} = \{(J_1, \{\alpha\})\}$, $F_{A_2} = \{(J_1, \{\beta\})\}$, $F_{A_3} = \{(J_1, \{\alpha, \beta\})\}$, $F_{A_4} = \{(J_2, \{\beta\})\}$, $F_{A_5} = \{(J_2, \{\gamma\})\}$, $F_{A_6} = \{(J_2, \{\beta, \gamma\})\}$, $F_{A_7} = \{(J_1, \{\alpha\}), (J_2, \{\beta\})\}$, $F_{A_8} = \{(J_1, \{\alpha\}), (J_2, \{\gamma\})\}$, $F_{A_9} = \{(J_1, \{\alpha\}), (J_2, \{\beta, \gamma\})\}$, $F_{A_{10}} = \{(J_1, \{\beta\}), (J_2, \{\beta\})\}$, $F_{A_{11}} = \{(J_1, \{\beta\}), (J_2, \{\gamma\})\}$, $F_{A_{12}} = \{(J_1, \{\beta\}), (J_2, \{\beta, \gamma\})\}$, $F_{A_{13}} = \{(J_1, \{\alpha, \beta\}), (J_2, \{\beta\})\}$, $F_{A_{14}} = \{(J_1, \{\alpha, \beta\}), (J_2, \{\gamma\})\}$, $F_{A_{15}} = F_A$, $F_{A_{16}} = F$. Soft open sets $(\tilde{\tau}) = \{F_A, F, F_{A_2}, F_{A_3}, F_{A_{11}}, F_{A_{12}}, F_{A_{14}}\}$, Soft closed sets $(\tilde{\tau})^C = \{F_A, F, F_{A_9}, F_{A_6}, F_{A_7}, F_{A_1}, F_{A_4}\}$. Then $(F_A, \tilde{\tau})$ is a soft topological space , and $Sg^{**}\psi$ - closed sets= $\{F_A, F, F_{A_1}, F_{A_4}, F_{A_5}, F_{A_6}, F_{A_7}, F_{A_8}, F_{A_9}\}$.

Theorem: 3.3

I. Each soft closed set is also a $Sg^{**}\psi$ - closed set.
II. Each soft g**- closed set is $Sg^{**}\psi$ - closed set.
III. Each soft α - closed set is also a $Sg^{**}\psi$ - closed set.
IV. Each soft ψ - closed set is also a $Sg^{**}\psi$ - closed set.
V. Each soft semi generalized closed set is also a $Sg^{**}\psi$ - closed set

Proof:

(I) and (II) ⇒ follows from the fact that "Every soft closed set is a soft ψ-closed set".

(III) ⇒ follows from the fact that " Every soft α – closed set is ψ- closed set".

(IV) ⇒ follows from the fact that " Every soft generalized*-open set is soft generalized open set".

(V) ⇒ follows from the fact that " Every soft semi open set is soft generalized*-open set ".

However, the converse of the above theorem need not be valid, which can be seen from the following examples.

Example 3.4

I. Let us take (Example 3.2), here $F_{A_8} = \{(J_1, \{\alpha\}), (J_2, \{\gamma\})\}$ and $F_{A_5} = \{(J_2, \{\gamma\})\}$ are Sg^{**} ψ - closed sets but not soft closed sets.

II. Let U={α, β, γ, δ}, $E = \{J_1, J_2, J_3\}$ and $A = \{J_1, J_2\}E$. $F_A = \{(J_1, \{\alpha, \beta\}), (J_2, \{\delta\})\}$, $F_{A_1} = \{(J_1, \{\alpha\})\}$, $F_{A_2} = \{(J_1, \{\beta\})\}$, $F_{A_3} = \{(J_1, \{\alpha, \beta\})\}$, $F_{A_4} = \{(J_1, \{\alpha\}), (J_2, \{\delta\})\}$, $F_{A_5} = \{(J_1, \{\beta\}), (J_2, \{\delta\})\}$, $F_{A_6} = \{(J_2, \{\delta\})\}$, $F_{A_7} = F_{A', F_{A_8} = F}$. Soft open sets $(\widetilde{\tau}) = \{F_A, F, F_{A_5}, F_{A_2}\}$, Soft closed sets $(\widetilde{\tau})^C = \{F_A, F_\emptyset, F_{A_1}, F_{A_4}\}$. Then $\left(F_A, \widetilde{\tau}\right)$ is a soft topological space. Sg^{**} ψ - closed sets= $\{F_A, F, F_{A_1}, F_{A_3}, F_{A_4}, F_{A_6}\}$. Here $F_{A_6} = \{(e_2, \{\delta\})\}$ is Sg^{**} ψ - closed set but not soft generalized**-closed set.

III. Let us take example 3.4 (II). soft α-closed sets = $\{F_A, F, F_{A_1}, F_{A_4}, F_{A_6}\}$. Here $F_{A_3} = \{(e_1, \{\alpha, \beta\})\}$ is Sg^{**} ψ - closed set but it is not soft α-closed set.

IV. Let us take example 3.4 (II). soft ψ-closed sets = $\{F_A, F, F_{A_1}, F_{A_4}, F_{A_6}\}$. Here $F_{A_3} = \{(e_1, \{\alpha, \beta\})\}$ is Sg^{**} ψ - closed set but it is not soft ψ-closed set.

V. Let us take example 3.4 (II). soft semi generalized-closed sets $= \{F_A, F, F_{A_1}, F_{A_4}, F_{A_6}\}$. Here $F_{A_3} = \{(e_1, \{\alpha, \beta\})\}$ is Sg^{**} ψ - closed set but it is not soft semi generalized closed set.

Remark: 3.5

I. Soft generalized semi-closed sets and Sg^{**} ψ are mutually interdependent.
II. Soft ψ generalized-closed sets and Sg^{**} ψ are mutually interdependent.
III. Soft g* ψ-closed sets and Sg^{**} ψ are mutually interdependent.
IV. soft $\psi\hat{g}$ -closed sets and Sg^{**} ψ are mutually interdependent.

The mutual interdependence of the aforementioned sets is substantiated by the following example.

Example: 3.6

- Let us take Example 3.4 (II). Here soft generalized semi closed sets and Sg^{**} ψ-closed sets are $\{F_A, F, F_{A_1}, F_{A_3}, F_{A_4}, F_{A_6}\}$.
- Let $U = \{\alpha, \beta, \gamma, \delta\}$, $E = \left\{J_1, J_2, J_3\right\}$ and $A = \left\{J_2, J_3\right\}E$, $F_A = \{(J_2, \{\alpha\}), (J_3, \{\beta, \gamma\})\}$,

$F_{A_1} = \{(J_2, \{\alpha\})\}$, $F_{A_2} = \{(J_2, \{\alpha\}), (J_3, \{\beta\})\}$, $F_{A_3} = \{(J_2, \{\alpha\}), (J_3, \{\gamma\})\}$,
$F_{A_4} = F_A$
$F_{A_5} = \{(J_3, \{\beta\})\}$, $F_{A_6} = \{(J_3, \{\gamma\})\}$, $F_{A_7} = \{(J_3, \{\beta, \gamma\})\}$, $F_{A_8} = F$. Soft open sets $(\widetilde{\tau}) = \{F_A, F\ , F_{A_2}, F_{A_7}\}$, Soft closed sets $(\widetilde{\tau})^C = \{F_A, F\ , F_{A_1}, F_{A_3}\}$. Then $(F_A, \widetilde{\tau})$ is a soft topological space. $Sg^{**}\psi$ -closed sets = soft ψg-closed sets = soft g* ψ-closed sets = soft $\psi\hat{g}$ -closed sets = $\{F_A, F\ , F_{A_1}, F_{A_3}, F_{A_6}\}$.

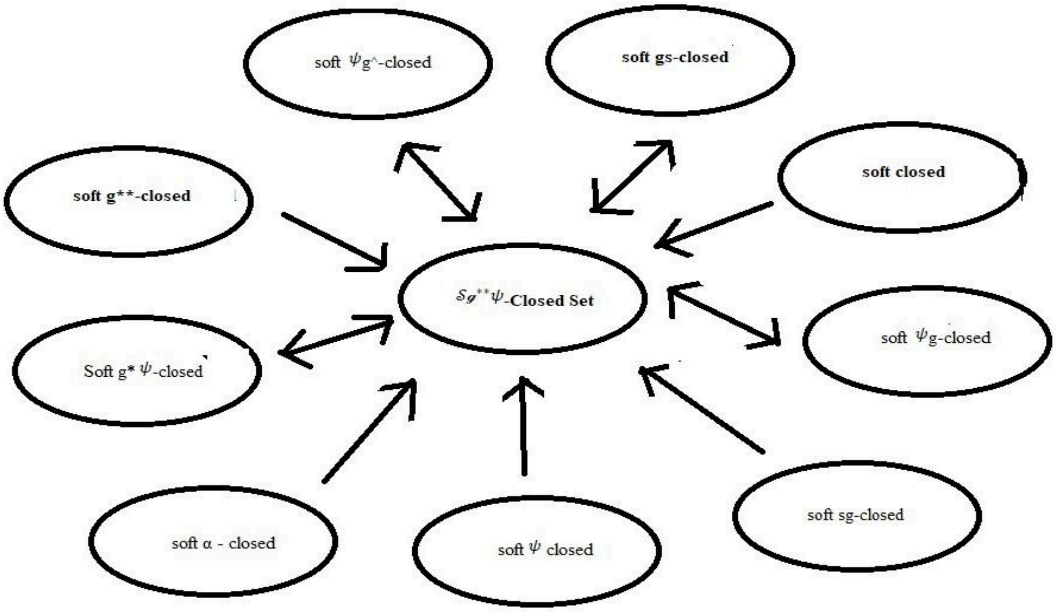


Figure-1 The diagram depicts the implication structure connecting the existing classes of soft closed sets with the newly proposed class of $Sg^{**}\psi$ -closed sets.

Theorem: 3.7

If F_A is $Sg^{**}\psi$ -closed set in $(F_A, \widetilde{\tau})$ and $F_A G_A \psi cl(F_A)$ then G_A is $Sg^{**}\psi$ -closed set.

Proof:

Suppose that F_A is $Sg^{**}\psi$ -closed set in $(F_A, \widetilde{\tau})$ and $F_A G_A \psi cl(F_A)$. Let $G_A U_A$ and U_A is soft g*-open set in $(F_A, \widetilde{\tau})$. Since $F_A G_A$ and $G_A U_A F_A U_A$. Hence $\psi cl(F_A)\ (U_A)$ [Since F_A is $Sg^{**}\psi$ -closed]. $\Rightarrow$ $G_A \psi cl(F_A)$. $\Rightarrow$ $G_A U_A$ and U_A is soft g*-open set in $(F_A, \widetilde{\tau})$. Therefore G_A is $Sg^{**}\psi$ -closed set.

Theorem:3.8

Arbitrary union of $Sg^{**}\psi$ -closed set is $Sg^{**}\psi$ - closed set.

Proof:

Suppose that F_A and G_A are two arbitrary $Sg^{**}\psi$ -closed sets . Let $F_A \cup G_A U_A$ is soft g*-open set. ⇒ $F_A U_A$ and $G_A U_A$. We know that U_A is soft g*-open set in $\left(F_A, \tilde{\tau}\right)$ and F_A and G_A are $Sg^{**}\psi$ - closed sets $\psi cl(F_A)U_A$ and $\psi cl(G_A)U_A$. Therefore $\psi cl\{(F_A \cup G_A)\} = \psi cl(F_A)\cup\psi cl(G_A)U_A$.

Theorem: 3.9

If a set F_A is $Sg^{**}\psi$ -closed set in $\left(F_A, \tilde{\tau}\right)$ if and only if $\psi cl(F_A)\setminus(F_A)$ contains only null soft closed set.

Proof:

Suppose that F_A is $Sg^{**}\psi$-closed set in $\left(F_A, \tilde{\tau}\right)$. Let G_A be soft closed set and $G_A\psi cl(F_A)\setminus(F_A)$. Since G_A is soft closed and its relative complement G'_A is soft open. Now $G_A\psi cl(F_A)\setminus(F_A)G_A\,\psi cl(F_A)$ and $G_A F_A$. $G_A\psi cl(F_A)$ and $G_A F'_A$. Hence $F_A G'_A$. Consequently $\psi cl(F_A)G'_A$. [Since F_A is $Sg^{**}\psi$ -closed set]. Therefore $G_A\psi cl(F'_A)$. $G_A = \psi\, cl\left(F_A\right)\left(F'_A\right) = \emptyset$. $G_A = \emptyset$. Therefore $\psi cl(F_A)\setminus(F_A)$ contains only null soft closed set.

Conversely, $\psi cl(F_A)\setminus(F_A)$ contains only null soft closed set.

Since $\psi cl(F_A)\setminus(F_A) = \emptyset$. $\psi cl(F_A) = F_A F_A$ is soft closed set. We know that every soft closed set is $Sg^{**}\psi$ - closed set . Therefore F_A is $Sg^{**}\psi$ - closed set.

Theorem: 3.10

A $Sg^{**}\psi$ - closed set F_A is soft closed if and only if $\psi cl(F_A)\setminus(F_A)$ is soft closed.

Proof:

If F_A is soft closed. $cl(F_A) = F_A$. Therefore $\psi cl\left(F_A\right)\setminus\left(F_A\right) = \emptyset$ and is soft closed set. Conversely, Suppose that $\psi cl(F_A)\setminus(F_A)$ is soft closed, Now by theorem 3.9, If a set F_A is $Sg^{**}\psi$ -closed set in $\left(F_A, \tilde{\tau}\right)$ if and only if $\psi cl(F_A)\setminus(F_A)$ contains only null soft closed set. Therefore F_A is soft closed.

Remark: 3.11

The intersection of any two $Sg^{**}\psi$ - closed sets in a soft topological space $\left(F_A, \tilde{\tau}\right)$ is a $Sg^{**}\psi$ - closed set.

IV – SOFT GENERALIZED**Ψ- OPEN SETS IN SOFT TOPOLOGICAL SPACES

In this section, we introduce the concepts of soft generalized **Ψ-open sets and soft generalized **Ψ-neighborhoods, and investigate their fundamental properties.

Definition: 4.1

A soft set F_A is called a soft generalized** ψ - open (briefly $Sg^{**}\psi$ -open)set in a soft topological space $\left(F_A, \widetilde{\tau}\right)$, if the relative complement F'_A is soft g** ψ -closed set in $\left(F_A, \widetilde{\tau}\right)$. Equivalently, A soft set F_A $is\ Sg^{**}\psi$, if $U_A\ \psi int(F_A)$ whenever $U_A F_A$ and U_A is soft g*-closed set.

Example: 4.2

Lets us take (example 3.2), U={ α, β, γ}, $E = \{J_1, J_2, J_3\}$ and $A = \{J_1, J_2\}E$, $F_A = \{(J_1, \{\alpha, \beta\}), (J_2, \{\beta, \gamma\})\}$. Soft open sets $(\widetilde{\tau}) = \{F_A, F, F_{A_2}, F_{A_3}, F_{A_{11}}, F_{A_{12}}, F_{A_{14}}\}$, Soft closed sets $(\widetilde{\tau})^C = \{F_A, F, F_{A_9}, F_{A_6}, F_{A_7}, F_{A_1}, F_{A_4}\}$. Then $(F_A, \widetilde{\tau})$ is a soft topological space , and $Sg^{**}\psi$ – open sets= $\{F_A, F, F_{A_2}, F_{A_3}, F_{A_5}, F_{A_{10}}, F_{A_{11}}, F_{A_{12}}, F_{A_{13}}, F_{A_{14}}\}$.

Theorem: 4.3

Each soft open set is also a $Sg^{**}\psi$ - open set

Proof:

Let F_A be a soft open set in $\left(F_A, \widetilde{\tau}\right)$ and let U_A be a soft g*-closed set in $\left(F_A, \widetilde{\tau}\right)$ such that $F_A U_A$. Since F_A is soft open set , $int(F_A) = F_A$. $int(F_A) = F_A U_A$ and since “Every soft open set is soft ψ-open set”. Therefore $int(F_A) \psi int(F_A) U_A$. $\psi int(F_A) U_A$ and U_A is soft g*- closed set in $\left(F_A, \widetilde{\tau}\right)$. Hence F_A is soft g**- open set.

The converse of the above theorem need not be valid, which can be seen from the following example.

Remark: 4.4

Lets us take (example 3.2), U={ α, β, γ}, $E = \{J_1, J_2, J_3\}$ and $A = \{J_1, J_2\}E$, $F_A = \{(J_1, \{\alpha, \beta\}), (J_2, \{\beta, \gamma\})\}$. Soft open sets $(\widetilde{\tau}) = \{F_A, F, F_{A_2}, F_{A_3}, F_{A_{11}}, F_{A_{12}}, F_{A_{14}}\}$, Soft closed sets $(\widetilde{\tau})^C = \{F_A, F, F_{A_9}, F_{A_6}, F_{A_7}, F_{A_1}, F_{A_4}\}$. Then $(F_A, \widetilde{\tau})$ is a soft topological space , and $Sg^{**}\psi$ –open sets= $\{F_A, F, F_{A_2}, F_{A_3}, F_{A_5}, F_{A_{10}}, F_{A_{11}}, F_{A_{12}}, F_{A_{13}}, F_{A_{14}}\}$. Here $F_{A_5}, F_{A_{10}}, F_{A_{13}}$ are $Sg^{**}\psi$- open sets but not soft open sets.

Theorem: 4.5

If F_A is $Sg^{**}\psi$ -open set in $(F_A, \widetilde{\tau})$ and $\psi int(F_A) G_A F_A$, then G_A is $Sg^{**}\psi$ - open set.

Proof:

Suppose that F_A is soft Sg^{**}ψ -open set in $\left(F_A, \tilde{\tau}\right)$ and $\psi int\left(F_A\right) G_A F_A$. Let $H_A G_A$ and H_A is soft g*-closed set in $\left(F_A, \tilde{\tau}\right)$. Since $G_A F_A$ and $H_A G_A$, we have $H_A F_A$. Since F_A is Sg^{**}ψ - open. $H_A \psi int\left(F_A\right)$, also since $\psi int\left(F_A\right) G_A$ H_A $\psi int\left(F_A\right) \psi int\left(G_A\right)$

$H_A \psi int(G_A)$. Therefore G_A is Sg^{**}ψ -open set.

Theorem: 4.6

Finite intersection of Sg^{**}ψ -open sets are Sg^{**}ψ -open set.

Proof:

Suppose that F_A and G_A are Sg^{**}ψ -open sets . Let $U_A F_A \cap G_A$ and U_A is soft g*-closed set in $\left(F_A, \tilde{\tau}\right)$. Since $U_A F_A \cap G_A$, $U_A F_A$ and $U_A G_A$. We know that U_A is soft g*-closed set in $\left(F_A, \tilde{\tau}\right)$ and F_A and G_A are soft g**-open sets $U_A \psi int(F_A)$ *and* $U_A \psi int(G_A)$. Therefore $U_A \psi int(F_A) \cap \psi int(G_A)$

Definition: 4.7

Let $\left(F_A, \tilde{\tau}\right)$ be a soft topological space, $F_B F_A$ and $\in F_B$. If there exists a Sg^{**}ψ - open set F_C such that $\alpha \in F_C F_B$ then α is called a Sg^{**}ψ - interior point of F_B and the soft union of all Sg^{**}ψ -interior points of F_B is denoted by $Sg^{**}\psi - int(F_B)$.

Definition: 4.8

Let A $\left(F_A, \tilde{\tau}\right)$ be a soft topological space, $F_B F_A$ and $\alpha \in F_A$. If there exists a soft Sg^{**}ψ -open set F_C such that $\alpha \in F_C F_B$ then F_B is called a Sg^{**}ψ -neighbourhood of α. Set of all Sg^{**}ψ -neighbourhoods of α is denoted by $N(\alpha)$, is called family of Sg^{**}ψ - neighbourhoods of α that is $N(\alpha) = \{F_B : F_C \in \tilde{\tau}\}$ and $\alpha \in F_C F_B$. In particular, $V(\alpha) = \{F_C \in \tilde{\tau} : \alpha \in F_C\}$.

Example: 4.9

Let U={α, β, γ, δ}, $E = \{J_1, J_2, J_3\}$ and $A = \{J_1, J_2\} E$. $F_A = \{(J_1, \{\alpha, \beta\}), (J_2, \{\delta\})\}$, $F_{A_1} = \{(e_1, \{\alpha\})\}$, $F_{A_2} = \{(e_1, \{\beta\})\}$, $F_{A_3} = \{(e_1, \{\alpha, \beta\})\}$, $F_{A_4} = \{(e_1, \{\alpha\}), (e_2, \{\delta\})\}$, $F_{A_5} = \{(e_1, \{\beta\}), (e_2, \{\delta\})\}$, $F_{A_6} = \{(e_2, \{\delta\})\}$, $F_{A_7} = F_A$, $F_{A_8} = F$. Soft open sets $(\tilde{\tau}) = \{F_A, F, F_{A_5}, F_{A_2}\}$, Soft closed sets $(\tilde{\tau})^C = \{F_A, F_\emptyset, F_{A_1}, F_{A_4}\}$. Then $\left(F_A, \tilde{\tau}\right)$ is a soft topological space. Sg^{**}ψ - closed sets= $\{F_A, F, F_{A_1}, F_{A_3}, F_{A_4}, F_{A_6}\}$ and Sg^{**}ψ - open sets $= \{F_A, F, F_{A_2}, F_{A_5}\}$

Let (i) $\alpha_1 = \{(e_1, \{a\})\}$. Then Sg^{**}ψ -neighbourhoods $\tilde{N}(\alpha_1) = \{F_A\}$ and Sg^{**}ψ open neighbourhoods $\tilde{V}(\alpha_1) = \{F_A\}$.

(ii) $\alpha_2 = \{(e_1, \{b\})\}$. Then $Sg^{**}\psi$ neighbourhoods $\widetilde{N}(\alpha_2) = \{F_A, F_{A_3}, F_{A_5}\}$ and $Sg^{**}\psi$ open neighbourhoods $\widetilde{V}(\alpha_2) = \{F_A\}$..

(iii) $\alpha_3 = \{(e_1, \{a, b\})\}$. Then $Sg^{**}\psi$ neighbourhoods $\widetilde{N}(\alpha_3) = \{F_A\}$ and $Sg^{**}\psi$ open neighbourhoods $\widetilde{V}(\alpha_3) = \{F_A\}$.

(iv) $\alpha_4 = \{(e_1, \{a\}), (e2, \{d\})\}$. Then $Sg^{**}\psi$ neighbourhoods $\widetilde{N}(\alpha_4) = \{F_A\}$ and $Sg^{**}\psi$ open neighbourhoods $\widetilde{V}(\alpha_4) = \{F_A\}$.

(v) $\alpha_5 = \{(e_1, \{b\}), (e2, \{d\})\}$. Then $Sg^{**}\psi$ neighbourhoods $\widetilde{N}(\alpha_5) = \{F_A\}$ and $Sg^{**}\psi$ open neighbourhoods $\widetilde{V}(\alpha_5) = \{F_A\}$.

(vi) $\alpha_6 = \{(e_2, \{d\})\}$. Then $Sg^{**}\psi$ neighbourhoods $\widetilde{N}(\alpha_6) = \{F_A, F_{A_5}\}$ and $Sg^{**}\psi$ open neighbourhoods $\widetilde{V}(\alpha_6) = \{F_A, F_{A_5}\}$.

Remark : 4.10

A $Sg^{**}\psi$ - neighbourhood generally need not be $Sg^{**}\psi$ -open set. It is proved by the given example below.

By example(4.9), Let U={α, β, γ, δ}, $E = \{J_1, J_2, J_3\}$ and $A = \{J_1, J_2\}E$. $F_A = \{(J_1, \{\alpha, \beta\}), (J_2, \{\delta\})\}$, Soft open sets $(\widetilde{\tau}) = \{F_A, F\ , F_{A_5}, F_{A_2}\}$, Soft closed sets $(\widetilde{\tau})^C = \{F_A, F_\emptyset, F_{A_1}, F_{A_4}\}$. Then $\left(F_A, \widetilde{\tau}\right)$ is a soft topological space. $Sg^{**}\psi$ - closed sets $= \{F_A, F\ , F_{A_1}, F_{A_3}, F_{A_4}, F_{A_6}\}$ and $Sg^{**}\psi$ - open sets= $\{F_A, F\ , F_{A_2}, F_{A_5}\}$. Let us take the $Sg^{**}\psi$ - neighbourhoods of F_{A_2} are $N(\alpha) = \{F_A, F_{A_3}, F_{A_5}\}$. Here F_{A_3} is not a $Sg^{**}\psi$ - open set.

$\alpha = F_{A_2} = \{(e_1, \{b\})\}$. Then soft g**-neighbourhoods $N(\alpha) = \{F_A, F_{A_3}, F_{A_5}\}$ but it is not a soft g**-open set because F_{A_3} is not a soft g**-open set.

Theorem : 4.11

Each soft neighbourhood is also a $Sg^{**}\psi$ -neighbourhood

Proof:

Let α be a soft neighbourhood of a soft set $F_B(F_A, \widetilde{\tau})$, then there exists a soft open set F_C such that $\Rightarrow$ $\alpha \in F_C F_B$. As we know that Every soft open set is $Sg^{**}\psi$ -open set such that $\alpha \in F_C F_B$. Hence α is $Sg^{**}\psi$ -neighbourhood of F_A.

V-REPRESENTATION OF Sg^{**}ψ - CLOSED SETS IN SOFT TOPOLOGY AND THEIR OPERATIONS USING BIPARTITE GRAPHS

In this section, we present a representation of soft Sg^{**}ψ-closed sets using a bipartite graph and discuss the operations on soft Sg^{**} ψ - closed sets.

Definition : 5.1[4]

Every soft set can be graphically represented with the help of bipartite graph by taking partite sets A and X where A is the subset of parameter set and X is the universe.

Example 5.2[4]

X={ X_1, X_2, X_3, X_4, X_5, X_6, X_7}, E={ E_1, E_2, E_3, E_4, E_5}, A={ E_1, E_2, E_3} $\subseteq$ E

F_A ={(E_1,{ X_1, X_4, X_5}), (E_2,{ X_2, X_3, X_6}), (E_3,{ X_3, X_6})}

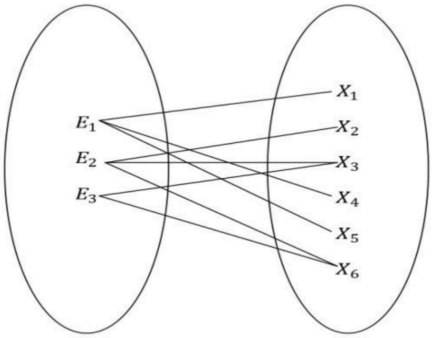


Figure 2. Soft set represented by bipartite graph”

Definition 5.3

Every Sg^{**} ψ -closed set can be graphically represented with the help of bipartite graph by taking partite sets A and X where A is the subset of parameter set and X is the universe.

Example 5.4

Let U={ α, β, γ}, $E = \{J_1, J_2, J_3\}$ and $A = \{J_1, J_2\}E$, $F_A = \{(J_1, \{\alpha, \beta\}), (J_2, \{\beta, \gamma\})\}$ $F_{A_1} = \{(J_1, \{\alpha\})\}$, $F_{A_2} = \{(J_1, \{\beta\})\}$, $F_{A_3} = \{(J_1, \{\alpha, \beta\})\}$, $F_{A_4} = \{(J_2, \{\beta\})\}$, $F_{A_5} = \{(J_2, \{\gamma\})\}$, $F_{A_6} = \{(J_2, \{\beta, \gamma\})\}$, $F_{A_7} = \{(J_1, \{\alpha\}), (J_2, \{\beta\})\}$, $F_{A_8} = \{(J_1, \{\alpha\}), (J_2, \{\gamma\})\}$, $F_{A_9} = \{(J_1, \{\alpha\}), (J_2, \{\beta, \gamma\})\}$, $F_{A_{10}} = \{(J_1, \{\beta\}), (J_2, \{\beta\})\}$, $F_{A_{11}} = \{(J_1, \{\beta\}), (J_2, \{\gamma\})\}$, $F_{A_{12}} = \{(J_1, \{\beta\}), (J_2, \{\beta, \gamma\})\}$, $F_{A_{13}} = \{(J_1, \{\alpha, \beta\}), (J_2, \{\beta\})\}$, $F_{A_{14}} = \{(J_1, \{\alpha, \beta\}), (J_2, \{\gamma\})\}$, $F_{A_{15}} = F_{A'}$, $F_{A_{16}} = F$. Soft open sets $(\tilde{\tau}) = \{F_A, F\ , F_{A_2}, F_{A_3}, F_{A_{11}}, F_{A_{12}}, F_{A_{14}}\}$, Soft closed sets

$(\tilde{\tau})^{C} = \{ F_{A}, F, F_{A_9}, F_{A_6}, F_{A_7}, F_{A_1}, F_{A_4} \}$. Then $(F_{A}, \tilde{\tau})$ is a soft topological space , and $Sg^{**}\psi$ -closed sets= $\{ F_{A}, F, F_{A_1}, F_{A_4}, F_{A_5}, F_{A_6}, F_{A_7}, F_{A_8}, F_{A_9} \}$.

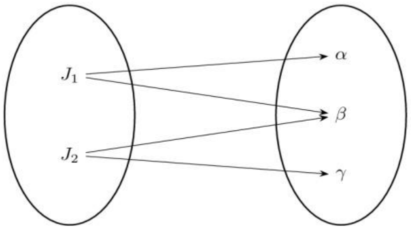


Figure 3. $Sg^{**}\psi$ closed-set represented by bipartite graph”

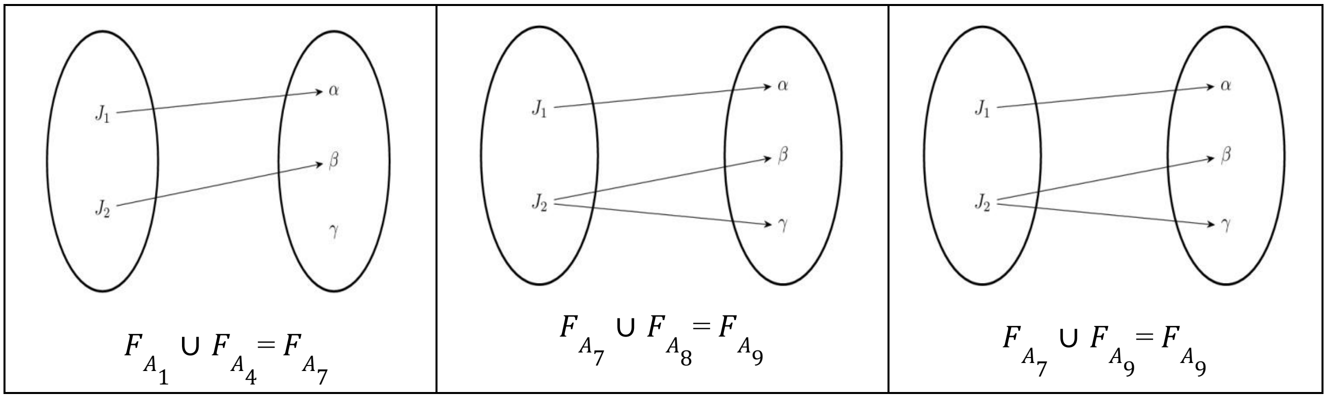
“**Property 5.5**

The union of any two $Sg^{**}\psi$ -closed sets admits a representation in terms of a bipartite graph. Consider the example 5.4 Here $Sg^{**}\psi$ -closed sets are $\{ F_{A}, F, F_{A_1}, F_{A_4}, F_{A_5}, F_{A_6}, F_{A_7}, F_{A_8}, F_{A_9} \}$. Here, we represent certain $Sg^{**}\psi$ unions of the $Sg^{**}\psi$ -closed sets.

$F_{A_1} \cup F_{A_4} = \{(J_1, \{\alpha\}), (J_2, \{\beta\})\} = F_{A_7}$ is also a $Sg^{**}\psi$ -closed set.

$F_{A_7} \cup F_{A_8} = \{(J_1, \{\alpha\}), (J_2, \{\beta, \gamma\})\} = F_{A_9}$ is also a $Sg^{**}\psi$ -closed set.

$F_{A_7} \cup F_{A_9} = \{(J_1, \{\alpha\}), (J_2, \{\beta, \gamma\})\} = F_{A_9}$ is also a $Sg^{**}\psi$ -closed set.



$F_{A_1} \cup F_{A_4} = F_{A_7}$

$F_{A_7} \cup F_{A_8} = F_{A_9}$

$F_{A_7} \cup F_{A_9} = F_{A_9}$

Figure 4. Union of $Sg^{**}\psi$ -closed set by bipartite graph

Property 5.6

Finite intersection of any two $Sg^{**}\psi$ -closed sets admits a representation in terms of a bipartite graph. Consider the above example 5.4 and we represent certain $Sg^{**}\psi$ intersections of the $Sg^{**}\psi$ -closed sets.

$F_{A_7}F_{A_9} = \{(J_1, \{\alpha\}), (J_2, \{\beta\})\} = F_{A_7}$ is also a $Sg^{**}\psi$ closed set.

$F_{A_8}F_{A_9} = \{(J_1, \{\alpha\}), (J_2, \{\gamma\})\} = F_{A_8}$ is also a $Sg^{**}\psi$ closed set.

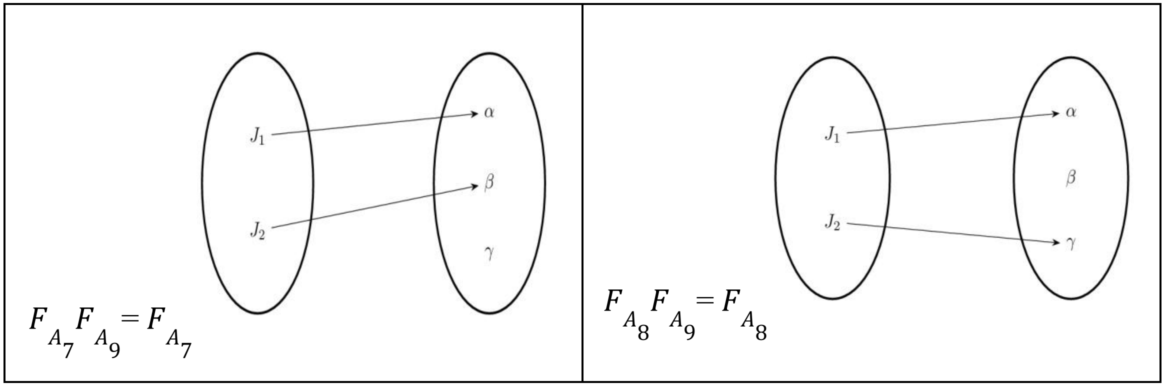


Figure 5. Intersection of $Sg^{**}\psi$ -closed sets by bipartite graph”

Property 5.7

The union operation involving two distinct $Sg^{**}\psi$ -closed sets can be illustrated as follows.

Let U=V={α, β, γ, δ}, $E = \{J_1, J_2, J_3\}$ and $A = \{J_1, J_2\}E$ and $B = \{J_2, J_3\}E$. $F_A = \{(J_1, \{\alpha, \beta\}), (J_2, \{\delta\})\}$, $F_B = \{(J_2, \{\alpha\}), (J_2, \{\beta, \gamma\})\}$, $Sg^{**}\psi$ - closed sets of $\left(F_A, \widetilde{\tau}\right) = \{ F_A, F\ , F_{A_1}, F_{A_3}, F_{A_4}, F_{A_6} \}$. $Sg^{**}\psi$ - closed sets of $\left(F_B, \widetilde{\tau}\right)$ =$\{ F_B, F\ , F_{B_1}, F_{B_3}, F_{B_6} \}$.

$F_A \,\widetilde{\boxtimes}\, F_B$=$\{F_A, F\ , F_{A_1}, F_{A_3}, F_{A_4}, F_{A_6}, F_B, F_{B_1}, F_{B_3}, F_{B_6}\}$.

Where $F_A = \{(J_1, \{\alpha, \beta\}), (J_2, \{\delta\})\}$, $F_{A_1} = \{(J_1, \{\alpha\})\}$, $F_{A_3} = \{(J_1, \{\alpha, \beta\})\}$, $F_{A_4} = \{(J_1, \{\alpha\}), (J_2, \{\delta\})\}$, $F_{A_6} = \{(J_2, \{\delta\})\}$, $F_B = \{(J_2, \{\alpha\}), (J_3, \{\beta, \gamma\})\}$, $F_{B_1} = \{(J_2, \{\alpha\})\}$, $F_{B_3} = \{(J_2, \{\alpha\}), (J_3, \{\gamma\})\}$, $F_{B_6} = \{(J_3, \{\gamma\})\}$

$\Rightarrow F_A \,\widetilde{\boxtimes}\, F_B$=$\{\left(J_1, \{\alpha, \beta\}\right), \left(J_2, \{\delta\}\right)\}, \{\left(J_2, \{\alpha\}\right), \left(J_3, \{\beta, \gamma\}\right)\}, \{(J_3, \{\gamma\})\}\}$ can be drawn by bipartite graph

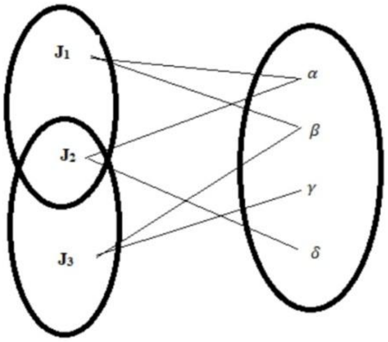


Figure 6. Union of two $Sg^{**}\psi$ -closed set by bipartite graph"

Property 5.8

The restricted union operation involving two distinct $Sg^{**}\psi$ -closed sets can be illustrated as follows from the above property 5.7

F_{A} $_{\mathrm{R}\,F\,\mathrm{B}} = \{(J_2, \{\alpha, \delta\})\}, \{(J_3, \{\beta, \gamma\})\}$ can be represented by bipartite graph as shown below

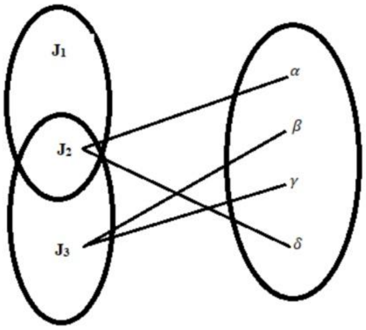


Figure 7. Restricted union of two $Sg^{**}\psi$ -closed set by bipartite graph

Property 5.9

The extended intersection operation involving two distinct $Sg^{**}\psi$ -closed sets can be illustrated as follows from the above property 5.7

$\Rightarrow$ F_{A} $\varepsilon_{F\mathrm{B}} = \{\{(J_1, \{\alpha, \beta\}), (J_2, \{\delta\})\}, F_B = \{(J_2, \{\alpha\}), (J_3, \{\beta, \gamma\})\}\}$ can be represented by bipartite graph as shown below

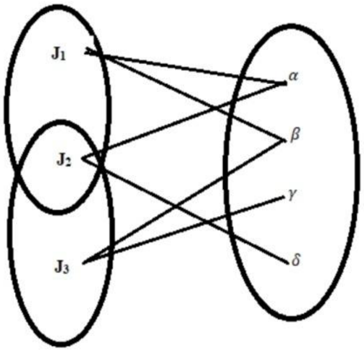


Figure 8. Extended Intersection of two $Sg^{**}\psi$ -closed set by bipartite graph”

Property 5.10

AND - operation involving two distinct $Sg^{**}\psi$ -closed sets can be illustrated as follows from the above property 5.7

$$F_A \wedge F_B \Rightarrow F_A \wedge \left\{F_{B'}, F_{B_1}, F_{B_3}, F_{B_6}\right\} = \{((J_1, J_2)\{\alpha\})\}$$

$$F_{A_1} \wedge \left\{F_{B'}, F_{B_1}, F_{B_3}, F_{B_6}\right\} = \{((J_1, J_2)\{\alpha\})\}$$

$$F_{A_3} \wedge \left\{F_{B'}, F_{B_1}, F_{B_3}, F_{B_6}\right\} = \{((J_1, J_2)\{\alpha\})\}$$

$$F_{A_4} \wedge \left\{F_{B'}, F_{B_1}, F_{B_3}, F_{B_6}\right\} = \{((J_1, J_2)\{\alpha\})\}$$

$$F_{A_6} \wedge \left\{F_{B'}, F_{B_1}, F_{B_3}, F_{B_6}\right\} = \{F\ \}$$

$$\Rightarrow F_A \wedge F_B = \{((J_1, J_2)\{\alpha\})\}.$$

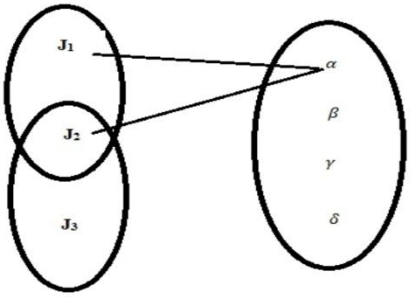


Figure 9. AND - Operation of two $Sg^{**}\psi$ -closed set by bipartite graph”

Property 5.11

OR - operation involving two distinct $Sg^{**}\psi$ -closed sets can be illustrated as follows from the above property 5.7

$F_A F_B$

$\Rightarrow F\,\mathrm{A}\{F_{B}, F_{B_1}, F_{B_3}, F_{B_6}\} = \{\{((J_1, J_2)\{\alpha, \beta\}), ((J_2, J_3), \{\beta, \gamma, \delta\})\}, \{((J_1, J_2)\{\alpha, \beta\}), (J_2, \{\delta\})\}, \{((J_1, J_2)\{\alpha, \beta\}), ((J_2, J_3), \{\gamma, \delta\})\}, \{((J_1, J_3)\{\alpha, \beta, \gamma\}), (J_2, \{\delta\})\}\}.$

$F_{A_1}\{F_{B}, F_{B_1}, F_{B_3}, F_{B_6}\} = \{\{(J_1, J_2)\{\alpha\}), (J_3)\{\beta, \gamma\})\}, \{(J_1, J_2)\{\alpha\})\}, \{(J_1, J_2)\{\alpha\}), (J_3)\{\gamma\}\}, \{((J_1, J_3)\{\alpha, \gamma\})\}\}.$

$F_{A_3}\{F_{B}, F_{B_1}, F_{B_3}, F_{B_6}\} = \{\{(J_1, J_2)\{\alpha, \beta\}), ((J_3)\{\beta, \gamma\})\}, \{((J_1, J_2)\{\alpha, \beta\})\}, \{\{((J_1, J_2)\{\alpha, \beta\}), (J_3), \{\gamma\}\}, \{((J_1, J_3)\{\alpha, \beta, \gamma\})\}\}.$

$F_{A_4}\{F_{B}, F_{B_1}, F_{B_3}, F_{B_6}\} = \{\{(J_1, J_2)\{\alpha\}), ((J_2, J_3), \{\beta, \gamma, \delta\})\}, \{(J_1, J_2)\{\alpha\}), ((J_2), \{\delta\})\}, \{(J_1, J_2)\{\alpha\}), ((J_2, J_3), \{\gamma, \delta\})\}, \{((J_1, J_3)\{\alpha, \gamma\}), (J_2, \{\delta\})\}\}.$

$F_{A_6}\{F_{B}, F_{B_1}, F_{B_3}, F_{B_6}\} = \{\{((J_1, J_2)\{\alpha, \delta\}), (J_3)\{\beta, \gamma, \delta\}\}, \{(J_2)\{\alpha, \delta\})\}, \{(J_2)\{\alpha, \delta\}), (J_3), \{\gamma\}, \{((J_2, J_3), \{\gamma, \delta\})\}\}.$

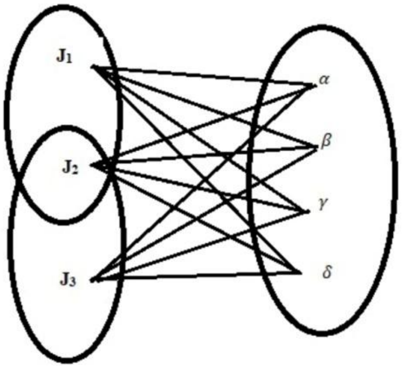


Figure 10. OR - Operation of two $Sg^{**}\psi$ -closed set by bipartite graph

VI-CONCLUSION

This paper introduced the concept of soft generalized** Ψ-closed sets in soft topological spaces and examined their basic properties and relationships with existing types of soft closed sets. It was observed that the proposed class extends several known notions. The study also included soft generalized** Ψ-open sets and their fundamental behavior. In addition, a bipartite graph representation was used to describe these sets and their operations in a clear and structured way. Further work can be carried out by extending these ideas to other frameworks such as fuzzy, intuitionistic fuzzy, and nano topological spaces. The study of related mappings and the use of these concepts in practical areas like decision-making and data analysis may also provide useful results. Exploring computational and graph-based approaches could offer additional developments in this area

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