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DARCY-BRINKMAN CONVECTION IN A JEFFREY FLUID SATURATED ROTATING ANISOTROPY POROUS MEDIUM WITH THERMODIFFUSION FOR CONVECTIVE DELIVERY SYSTEM

Sravan Nayek Gaikwad [a,*], Farisha Jabin [b], Dnyaneshwar Madhavrao Surwase [c]

[a, b] Department of Mathematics, Gulbarga University, Kalaburagi - 585 106, Karnataka, India

[c] Department of Mathematics, Dr. B. A. Technological University Lonere - Raigad – 402 103, Maharashtra, India

[a, *] sravangaikwad06@gmail.com, [b]farisha.jabink7@gmail.com, [c]dmsurwase@dbatu.ac.in

ABSTRACT

In this paper, the onset of convective instabilities in a Jeffrey fluid saturated with rotating anisotropic porous medium is investigated in the presence of thermodiffusion. The problem for the convection-enhanced delivery system with anisotropy in porous media is modeled using the Darcy-Brinkman-Jeffrey constitutive relation. Such configuration plays a significant role in many bio-medical applications for precise delivery of fluids in taking advantage of controlled porous convection. The linear stability analysis is performed using the classical normal mode technique and the stability parameter thermal Rayleigh number is determined. The results are depicted pictorially and compared with previous published outcomes. It is observed that the rotations, relative effect of permeability, and thermal anisotropy stabilize the system in both stationary and oscillatory convective modes. The Jeffrey fluid parameter, mechanical anisotropy, solute buoyancy and thermodiffusion destabilize the system in stationary mode and stabilize the system in oscillatory mode whereas effects are dual in case of Lewis number.

Keywords: Anisotropy, Convection, Darcy-Brinkman Model, Instability, Rotation

1 INTRODUCTION

In the era of expeditious innovations and technological advancement, the non-Newtonian fluids have been emerged as invaluable assets and capturing the attention of the researchers worldwide over the past two decades. Their versatility spans across diverse sectors of the technological landscape, rendering them indispensible in various industrial applications. However, the intricate characteristics of non-Newtonian fluids present significant challenges for characterization using conventional mathematical models such as the Navier-Stokes equations. By delving into the complex behaviour of Jeffrey fluids, this work explores their responses to effects of various factors such as anisotropy, rotations and thermodiffusion. Jeffrey fluids are particularly important in modeling physiological fluids seen in biological systems, accurately representing blood circulation with respect to stress retardation and relaxation time. From enhancing biomedical treatments to optimizing industrial processes, the study of Jeffrey fluid continues to offer promising avenues for advancing scientific knowledge and technological capabilities [1]. Nowadays, the impact of anisotropy of porous media on convective motion with rotations has made copious attraction of researchers due to its wide range of applications in fields of science, engineering and technology. Alex and Patil [2] were the first who have investigated the Influence of the anisotropy of porous media with rotation on the onset of convection. They reported that anisotropy in permeability makes the system unstable whereas anisotropy in thermal diffusivity has the opposite trend on the stability of the system. Govender and Vadasz [3] studied the effect of anisotropy of porous media on the convective stability driven by gravitational and Coriolis forces in the presence of thermal non-equilibrium. They reported that the rotation stabilizes convective motion in the porous system exhibiting anisotropy in permeability and thermal diffusivity of medium. Malashetty and Swamy [4] studied the effects anisotropy and rotation on the thresholds of convective movement applying linear and nonlinear hypotheses in a porous medium. It has been established that enhancement in the anisotropy of both mechanical and thermal diffusivity, hasten the convective motion for oscillatory mode. Influence of internal heating on the start of convective activity in rotation of anisotropic porous layer is measured by Bhadauria et al. [5] and shown that the anisotropic and rotation parameters diminish the heat transfer rate, and thus keep the convective motion under control. Haddad [6] investigated the results of mechanical anisotropy on the onset of stabilities in the rotating porous layer and he observed that the critical Rayleigh numbers obtained by linear stability theory and nonlinear stability theory using the energy method are identical.

Recently, Capone and Gentile [7] analyzed the influence of vertical rotations under local thermal non-equilibrium on the fluid convection in an anisotropic porous layer and proved the coincidence between local and global stability thresholds. Further, Yadav [8] studied the effects of gravity and rotational forces on the thermal convection of fluid saturated with anisotropic porous medium. He demonstrated that anisotropy in permeability advances the convective motion in fluid whereas the anisotropy in thermal diffusivity, variations in gravity, and rotations delay the onset of convective motion.

In the above discussed situations, almost all the occurring fluids show non-Newtonian in nature and their flows in rotating porous media has made special attraction of researchers in the recent past decades. For instance, in the process of extraction of crude oils from reservoirs the heavy crude is non-Newtonian type of the fluids and the rheology of such oils are demonstrated by Darcy model taking the non-Newtonian characters of fluids. Such model plays significant role in the study of fluid transportation management in the oil displacement mechanism for advancement the potential of oil revival. In the above literature, many of the researchers have taken into consideration various non-Newtonian fluids; therefore a particular one model is not sufficient to analyze the characteristics of all the non-Newtonian fluids. Consequently, the different types of fluid models are used by the researchers in the give literatures such as the LTNE, Oldroyd-B model, Maxwell model and Rivlin–Ericksen model.

In scenarios, Convection-Enhanced Delivery (CED) where the fluids are infused under pressure into tissues, the Jeffrey model is more effective to account the non-viscous behavior of the interstitial fluid, enabling a better understanding of drugs propagation throughout tumor tissue. Also it is often used to model blood as a viscoelastic fluid, offering better accuracy than simply Newtonian models scenarios with slip or porous boundaries. The present work contributes the study of stability of non-Newtonian fluid using modified Jeffrey model. This is a visco-elastic fluid model representing the effects of relaxation and retardation times. It has been proved that the Jeffrey model is a quite successful model handling the rheology of fluids in living body. For example, blood, saliva, gastric fluids, synovial, chime, etc [9–14]. The thermal convection of non-Newtonian fluid saturated porous media has been studied using Oldroyd–B, Maxwell, Kuvshiniski and Rivlin–Ericksen considerably [15–20]. In contrast, the rare attention has been found handling the problems on thermal convection in fluids with Jeffrey model. A few decades ago, the investigation on thermal instability of Visco-elastic fluid has been done by Martinez-Mardones and Perez-Garcia [21] using Jeffrey fluid model and reported that the model helps to improve the drawbacks of Maxwell model for convective motion in fluid. Santhosh et al. [22] discussed the Jeffrey fluid convection in a horizontal tube saturating porous matrix. It has been observed that the Jeffrey parameter and the effective viscosity improve the radius of escalating tube. Recently, Yadav [23] studied the influence of Coriolis force on thermal convection of Jeffrey fluid saturated with anisotropic porous medium using linear stability theory. He investigated that the rotation parameter and the thermal anisotropy affect the delay in beginning of fluid convective motion, whereas the Jeffery parameter and the mechanical anisotropy illustrate a dual effect on the convective system.

Motivating by the numerous industrial applications of Jeffery fluids as described in the literature review, the objective of the present work is to investigate the impacts of anisotropic situation in both permeability and thermal diffusivity on convective instability in rotating fluid saturated porous layer in the presence of Soret effect. Employing the linear stability theories, an expression of a parameter which measures the stability of the porous system is derived analytically and the outcomes are depicted pictorially. The comparative analysis is also done with previous published results.

2 MATHEMATICAL MODELLING

2.1 Physical Configuration

A horizontal porous layer confined between two parallel plates at distance *'d'* is considered. The layer is saturated by incompressible Jeffrey fluid with motion driven by buoyancy. The bottom and top parallel plates preserved at temperatures $T_0 + \Delta T$ and T_0 respectively. The corresponding concentrations are taken as $C_0 + \Delta C$ and C_0. The plates are rotating together with uniform rate $'\Omega'$ about the z-axis under the action of constant gravity $'\bar{g}'$, as shown in **Figure 1.**

2.2 Governing Equations

If the layer is heated from below, the Darcy-Benard convection under the assessment of Boussinesq approximation with uniform rotations will set in due to temperature and concentration differences (***see Figure 1***). The governing equations describing this situation under the influence of Soret effect are [9, 18, 23-25]:

$$\nabla \cdot \bar{q} = 0 \tag{1}$$

$$\left(\frac{\mu \bar{K}_m^{-1}}{1+\lambda}\right)\bar{q} = -\nabla P + \mu_e \nabla^2 \bar{q} + \rho \bar{g} + \frac{2\rho_0}{\varepsilon}\left(\bar{q} \times \bar{\Omega}\right) \tag{2}$$

$$\left(\rho C_p\right)_m \frac{\partial T}{\partial t} + \frac{1}{\varepsilon}\left(\rho C_p\right)_f (\bar{q} \cdot \nabla) T = \nabla \cdot \left(\bar{k}_m \cdot \nabla T\right) \tag{3}$$

$$\frac{\partial C}{\partial t} + \frac{1}{\varepsilon}(\bar{q} \cdot \nabla) C = -\nabla \cdot j \tag{4}$$

$$\rho = \rho_0 \left(1 - \alpha_T (T - T_0) + \alpha_C (C - C_0)\right) \tag{5}$$

where, $\bar{q} = (u, v, w)$ is velocity of Jeffrey fluid, μ is viscosity of the fluid, μ_e is effective viscosity of the fluid, $\bar{K}_m$ is the permeability vector of anisotropic porous layer, λ is the Jeffrey fluid parameter, P is the pressure, ρ is density of the fluid, $\bar{g} = (0, 0, -g)$ is the acceleration due to gravity, ε is porosity of the layer, $\bar{\Omega}$ is the Coriolis force (rotational force), $\left(\rho C_p\right)_m$ is the heat capacity of the medium, $\left(\rho C_p\right)_f$ is the heat capacity of the fluid, T is temperature of the fluid, $\bar{k}_m$ is the thermal conductivity vector, C is concentration of the fluid, j is the mass flux, α_T and α_C are the thermal and solutal coefficients respectively.

2.3 Mass flux

The mass flux 'j' is the rate of mass diffusion. In the fluids, it is given by the sum of two diffusion terms: Fickian diffusion and mass diffusion by the Soret effect (Thermodiffusion) [26-27]:

$$-j = \beta_D \left(\nabla C\right) + \frac{\left(\beta_D S\right)}{T_0}\left(\nabla T\right) \tag{6}$$

Where, β_D is Brownian diffusion coefficient and S is the Soret coefficient. On Substituting Equation (6) in Equation (4), yields:

$$\frac{\partial C}{\partial t} + \frac{1}{\varepsilon}(\bar{q} \cdot \nabla) C = \beta_D \nabla^2 C + \frac{\left(\beta_D S\right)}{T_0} \nabla^2 T \tag{7}$$

2.4 Basic State

The basic state of the fluid is assumed to be quiescent. The quantities of the basic state are given by,

$$\bar{q}_b = (0,\, 0,\, 0),\, T = T_b(z),\, C = C_b(z),\, P = P_b(z),\, \rho = \rho_b(z) \tag{8}$$

which satisfy the equations:

$$\bar{q}_b = (0,\, 0,\, 0),\;\; T_b(z) = 1 - z \text{ and } C_b(z) = 1 - z \tag{9}$$

2.5 Perturbed State

On the basic state we superimpose small perturbations in the form [9, 28]:

$$\bar{q} = \bar{q}_b + \bar{q}\,',\, T = T_b + T',\;\; C = C_b + C',\, P = P_b + P',\, \rho = \rho_b + \rho' \tag{10}$$

where dashes indicate perturbations. Introducing equation (10) into equations (1) – (3), (5) and (7), and using the basic state equation (9), it can be obtained:

$$\nabla \cdot \bar{q}\,' = 0 \tag{11}$$

$$\left(\frac{\mu \bar{K}_m^{-1}}{1+\lambda}\right)\bar{q}\,' = -\nabla P' + \mu_e \nabla^2 \bar{q}\,' + \rho_0\left(\alpha_T T' - \alpha_C C'\right) g + \frac{2\rho_0 \Omega}{\varepsilon}\left(\bar{q}\,' \times \bar{e}_z\right) \tag{12}$$

$$\gamma \frac{\partial T'}{\partial t} + \frac{1}{\varepsilon}(\bar{q}\,' \cdot \nabla) T' = w' + \nabla \cdot \left(\bar{\alpha}_m \cdot \nabla T'\right)\Delta C \tag{13}$$

$$\frac{\partial C'}{\partial t} + \frac{1}{\varepsilon}(\bar{q}\,' \cdot \nabla) C' = w' + \beta_D \nabla^2 C' + \left(\frac{\beta_D S}{T_0}\right)\nabla^2 T' \tag{14}$$

where, $\gamma = \dfrac{\left(\rho C_p\right)_m}{\left(\rho C_p\right)_f}$ is the ratio of heat capacities, $\bar{\alpha}_m = \dfrac{\bar{k}_m}{\left(\rho C_p\right)_f}$ is the thermal diffusivity vector. Introducing the non-dimensional parameters:

$$\begin{gathered}\left(x', y', z'\right) = d\left(x^*,\, y^*,\, z^*\right),\, t' = \frac{\gamma d^2}{\alpha_{mz}} t^*,\, \bar{q}\,' = \frac{\alpha_{mz}}{d}\,\bar{q}\,^*, \\ T' = \left(\Delta T\right) T^*,\;\; C' = \left(\Delta C\right) C^*,\;\; P' = \frac{\mu \alpha_{mz}}{K_{mz}}\, P^*\end{gathered} \tag{15}$$

Using equation (15) in equations (11) - (14), the non-dimensional form of the governing equations can be written as (dropping the asterisks for simplicity):

$$\left(\nabla_1^2\left(\frac{1}{1+\lambda}\nabla_H^2 + \nabla_2^2\right)+Ta\,\frac{\partial^2}{\partial z^2}\right)w=\nabla_1^2\left(Ra\nabla_H^2 T - Ra_s\nabla_H^2 C\right) \tag{16}$$

$$\left(\frac{\partial}{\partial t} - \eta\nabla_H^2 - \frac{\partial^2}{\partial z^2}\right)T + \frac{1}{\varepsilon}(\bar{q}\cdot\nabla)T = w \tag{17}$$

$$\left(\frac{1}{\gamma}\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2\right)C + \frac{1}{\varepsilon}(\bar{q}\cdot\nabla)C = w + Sr\nabla^2 T \tag{18}$$

where, $\xi=\frac{K_{mx}}{K_{mz}}$ is the anisotropy parameter of permeability, $\eta=\frac{\alpha_{mx}}{\alpha_{mz}}$ is the anisotropy parameter of thermal diffusivity, $Ta=\left(\frac{2\rho_0\,\Omega K_{mz}}{\varepsilon\,\mu}\right)^2$ is the Taylor's number, $Dc=\frac{\mu_e\,K_{mz}}{\mu\,d^2}$ is Darcy-Brinkman parameter, $Ra=\frac{\alpha_T\,\Delta T\,\rho_0 g\,K_{mz}d}{\alpha_{mz}\,\mu}$ is the thermal Rayleigh number, $Ra_s=\frac{\alpha_T\,\Delta C\,\rho_0 g\,K_{mz}d}{\alpha_{mz}\,\mu}$ is the solutal Rayleigh number, $Le=\frac{\alpha_{mz}}{\beta_D}$ is the Lewis number and $Sr=\frac{(\beta_D\,S)\Delta T}{(\alpha_{mz}T_0)\Delta C}$ is the Soret effect parameter, $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ is the Lapacian operator, $\nabla_H^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ is the horizontal Lapacian operator, $\nabla_1^2=\frac{1}{\xi(1+\lambda)} - Dc\nabla^2$, $\nabla_2^2=\frac{1}{\xi(1+\lambda)}\frac{\partial^2}{\partial z^2} - Dc\nabla^2$.

The set of equations (16)-(18) form a system of eigen-value problem in which Ra may be considered as the eigenvalue and it is solved for stress-free and isothermal boundary conditions:

$$w=\frac{\partial^2 w}{\partial z^2}=T=C=0,\quad \text{at}\quad z=0,1 \tag{19}$$

3 LINEAR STABILITY ANALYSIS

This section predicts the thresholds of stationary and oscillatory convective movements of the fluid in the system. The linear stability theory is a significant tool in the analysis of local non-linear stability process. An eigen-value problem given by the system of equations (16)–(18), subjected with boundary conditions (19) can be solved using the time-dependent periodic deviations in horizontal components. To make this study, we write the linear version of equations (16)–(18) by neglecting the nonlinear terms as:

$$\left(\nabla_1^2\left(\frac{1}{1+\lambda}\nabla_H^2 + \nabla_2^2\right) + Ta\,\frac{\partial^2}{\partial z^2}\right)w = \nabla_1^2\left(Ra\nabla_H^2 T - Ra_s\nabla_H^2 C\right) \tag{20}$$

$$\left(\frac{\partial}{\partial t} - \eta\nabla_H^2 - \frac{\partial^2}{\partial z^2}\right)T = w \tag{21}$$

$$\left(\frac{1}{\gamma}\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2\right)C = w + Sr\nabla^2 T \tag{22}$$

The normal modes of perturbations in the velocity w, the temperature T and the concentration C are assumed to show the horizontal periodicity as follow [28-29]:

$$\left(w,\, T,\, C\right) = \left(W,\, \Theta,\, \Phi\right)(z)\cdot\exp\left(\sigma t + i(lx + my)\right) \tag{23}$$

where, $\sigma = \omega_r + i\omega_i$ is the growth rate in the perturbations, ω is the frequency of the oscillations, a is the wave number in the horizontal directions.

Substituting equation (23) in equations (20) - (22), the stability equations are obtained as:

$$\left(\Delta_1^2\left(\Delta_2^2 - \frac{1}{\xi(1+\lambda)}a^2\right) + Ta\,\mathrm{D}^2\right)W + a^2\left(Ra\,\Delta_1^2\Theta - Ra_s\,\Delta_1^2\Phi\right) = 0 \quad (24)$$

$$W + \left(\mathrm{D}^2 - a^2\eta - \sigma\right)\Theta = 0 \tag{25}$$

$$Le\,W + Sr\,Le\left(\mathrm{D}^2 - a^2\right)\Theta - \left(\mathrm{D}^2 - a^2 - \frac{Le}{\gamma}\sigma\right)\Phi = 0 \tag{26}$$

where, $\mathrm{D} = \frac{d}{dz}$, $\Delta_1^2 = \frac{1}{\xi(1+\lambda)} - Dc\left(D^2 - a^2\right)$, $\Delta_2^2 = \frac{1}{\xi(1+\lambda)}D^2 - Dc\left(D^2 - a^2\right)^2$. The nontrivial solution of system of equations (24) – (26) is given by:

$$\begin{vmatrix} R_{11} & Ra\,R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{vmatrix} = 0 \tag{27}$$

From equation (27), the expression of stability parameter Ra can be obtained as:

$$Ra = \frac{R_{11}\left(R_{22}R_{33} - R_{32}R_{23}\right) + R_{13}\left(R_{21}R_{32} - R_{22}R_{31}\right)}{R_{12}\left(R_{21}R_{33} - R_{31}R_{23}\right)} \tag{28}$$

where, $R_{11} = -\dfrac{\left(1+\delta_6{}^2\right)\left(\delta_2{}^2 + \delta_1{}^2\delta_6{}^2\right) + \delta_4^2 Ta}{\delta_5{}^2}$,

$R_{12} = \dfrac{a^2\left(1+\delta_6{}^2\right)}{\delta_5}$,

$R_{13} = -\dfrac{a^2 Ra_s\left(1+\delta_6{}^2\right)}{\delta_5}$,

$R_{21} = 1$,

$R_{22} = -\left(\sigma + \delta_3^2\right)$,

$R_{23} = 0$,

$R_{31} = Le$,

$R_{32} = -Sr\,Le\,\delta_1^2$,

$R_{33} = \frac{Le}{\gamma}\sigma + \delta_1^2$,

$\delta_1{}^2 = \pi^2 + a^2$,

$\delta_2{}^2 = \pi^2 + \xi a^2$,

$\delta_3{}^2 = \pi^2 + \eta a^2$,

$\delta_4^2 = \pi^2\xi^2\left(1+\lambda\right)^2$,

$\delta_5^2 = \xi^2\left(1+\lambda\right)^2$,

$\delta_6^2 = \delta_1{}^2 Dc\,\xi\left(1+\lambda\right)$.

3.1 Stationary Convection

When the rate of growth in the perturbation, is real quantity, σ (i. e. $\sigma = \omega_r$), then the instabilities are obtained in stationary convection mode. If $\omega_r > 0$, the perturbations appeared in equation (23) grow with time. It means that the system is in unstable mode. Whereas, if $\omega_r < 0$, the system is in stable mode. The case of $\sigma = \omega_r = 0$, accord to marginal stability in the stationary convection mode. Applying the principal of exchange of stability i.e. for $\sigma = 0$ [28], one can obtain the following expression of thermal Rayleigh number for stationary convection at marginal state as:

$$Ra^{st} = \frac{-\delta_1^2\delta_3^2 R_{11} + R_{13}\left(R_{32} + \delta_3^2 R_{31}\right)}{\delta_1^2 R_{12}} \tag{29}$$

For regular fluid $(\lambda = 0)$ saturated isotropic porous media $(\xi = 1 \text{ and } \eta = 1)$in the absence of Soret effect$(i.e. \text{ when } Sr = 0)$, mass diffusion $(i.e.\ Ra_s = 0)$, rotational force $(Ta = 0)$, the Equation (29) is reduced to:

$$Ra^{st} = \frac{\left(\pi^2 + a^2\right)^2}{a^2} \tag{30}$$

In this case, the critical value of Rayleigh number is $Ra_c = 4\pi^2$occurs at $a_c = \pi$. These are the classical results of Rayleigh-Benard problem for clear fluids saturated isotropic porous media with free-free boundaries [30-31].

3.2 Oscillatory Convection

The oscillatory state of the system is represented by non-dimensional parameters viz. Rayleigh number Ra^{osc} and frequency of oscillations ω. For the oscillatory convection mode of the system, $\omega_r = 0$ and $\omega_i \neq 0$. Therefore, from equation (28), the frequency of oscillations ω_i (dropping subscript '*i*') and Rayleigh number Ra^{osc} are respectively given as:

$$\omega^2 = \frac{R_1 R_2 - R_3 R_4}{R_2 R_5} \tag{32}$$

$$Ra^{osc} = \frac{R_1 + \omega^2 R_2 R_3 R_5}{R_4^2 + \omega^2 R_5} \tag{33}$$

where, $R_1 = R_{13}R_{32} + \delta_3^2 R_{31} - \delta_1^2 \delta_3^2 R_{11}$, $R_2 = \frac{Le}{\gamma} R_{12}$, $R_3 = R_{31} + \delta_1^2 + \frac{Le}{\gamma}\delta_3^2$, $R_4 = \delta_1^2 R_{12}$,
$R_5 = -\frac{Le}{\gamma} R_{11}$,

4 RESULTS AND DISCUSSION

The convection-enhanced delivery system of rotating Jeffrey fluid saturating anisotropic porous medium is analyzed using linear theory to determine the stability of convection in stationary and oscillatory modes. The stability of convections helps to optimize the distribution of the therapeutic agents in biological systems. The effects of rotations describing Taylor number (*Ta*), permeability relative to cross-sectional area representing Darcy-Brinkman number (*Dc*), Jeffery fluid parameter (λ), mechanical and thermal anisotropy parameters (ξ and η), Lewis number (*Le*) and thermodiffusion processes representing Soret parameter (*Sr*) and solute buoyancy describing solutal Rayleigh number (*Ras*), on the onset of convective motion in Jeffery fluid are analyzed in the present study. The expressions of thermal Rayleigh number (*Ra*) given by

Equations (29) and (33) are obtained analytically for stationary and oscillatory convections respectively using normal mode technique. The range of leading parameters appeared in the study are considered as: Taylor number Ta (0 - 1000), Darcy-Brinkman number Dc (0 - 1000), Jeffery fluid parameter λ (0 - 1), anisotropy parameter of permeability ξ (0 - 1), anisotropy parameter of thermal diffusivity η (0 - 1), solute Rayleigh number Ra_S (0 - 100), Lewis number Le (1 - 1000) and Soret parameter Sr (0 - 100). The convective instabilities in the Jeffrey fluid rotating system has been observed through the stability curves **(Figures 2–9)**. The inner region of stability curves shows the unstable mode of the system and the outer region of stability curves represents the stable mode of the system. Moreover, the solid curves in the figures represent stationary convection and the dashed curves represent oscillatory convection. The system has stabilizing effect means the motion of fluid particles in convective delivery phenomenon is slowing down whereas the system has destabilizing effect means the motion of fluid particles in the phenomenon is speeding up.

Figure 2–4 describe the effect of rotations by Taylor number (Ta), the relative effect of permeability by Darcy-Brinkman number (Dc) and the effect of anisotropy of porous layer in thermal diffusivity by the parameters (η) respectively on the stability of the system. It is found that on escalating the value of Taylor number Ta, and Darcy-Brinkman number Dc, and anisotropy parameter of thermal diffusivity η, increasing the thermal Rayleigh numbers Ra in both stationary and oscillatory convections of the system. Thus, the impact of these parameters is to detain the beginning of convective motion in the fluid layer. This is for the cause that the rotations, relative effect of permeability and anisotropy of thermal diffusivity prevent the vertical movement, which, in turn, decay the disruption in the arrangement and stabilize the system in both stationary and oscillatory convection cases.

Figures 5–8 demonstrate the effects of Jeffrey fluid by the parameter λ, anisotropy of porous layer in permeability by the parameters ξ, solute Rayleigh number by the parameter Ra_S, Soret effect by the parameter Sr respectively on the stability of the Jeffrey fluid system. From the figures, it is noticed that the thermal Rayleigh number Ra decreases in both stationary and oscillatory convections with an increase in these parameters. Thus, these parameters have destabilizing effect on the Jeffrey fluid convection in a porous layer with anisotropy. **Figure 9** depict the effects Lewis number Le on the stability of the system. From the figure, it is noticed that the thermal Rayleigh number Ra decreases in case of stationary convection and it increases in case of oscillatory convection with an increase in the values of Lewis number Le. Hence, Le has dual stabilizing effect on the Jeffrey fluid convection in a porous layer with anisotropy.

The comparative study for different effects and models has shown in **Figure 10.** The stability curves for stationary convective mode of the system studied by Horton and Rogers [31], Kumar and Bhadauria [15], Yadav [23], Vandaz [24] with Present study have depicted in this figure. In the present study, the influence of rotations on Jeffrey fluid convection using Darcy-Brinkman Model in the presence of Soret diffusion is studied for anisotropic porous medium. In the absence of mass diffusion and viscous force (represented by Brinkman Model), the results of present study are reduced to results reported by Yadav [23]. For the non-Newtonian fluid (in the absence of Jeffrey fluid model), the stability curve is shifted to the curve presented by Vandaz [24]. The stability curve under the influence of rotations by Kumar and Bhadauria [15] is depicted for isotropic porous media. In the absence of all the effects studied by above cited researchers, the stability curve reduced to the curve for clear fluids reported by Hortan and Rogers [31]. It is noted that the comparison makes good agreement and thus verified the accuracy of model used in the present study.

6 CONCLUSION

The effect of vertical rotations on the Darcy-Brinkman convection phenomena in a horizontal Jeffrey fluid porous layer with anisotropy is explored in the presence of Soret diffusion. The linear part of the problem is studied executing the classical normal mode approach. The impacts of parameters which appear in the present study on stability and finite amplitude oscillatory motion of the system are discussed. The results demonstrate that Taylor number (Ta), Darcy-Brinkman number (Dc), and the thermal anisotropy parameters (η) have stabilizing appearance in both stationary and oscillatory convection modes of the system. Whereas, Jeffery fluid parameter (λ), mechanical anisotropy parameters (ξ), solute Rayleigh number (Ra_S), Soret parameter (Sr) have destabilizing effect in both stationary and oscillatory convection modes of the system. The Lewis number (Le) reported dual stabilizing effect on the system. The outcomes obtained in the study are supposed to play a significant role also in many industrial processes viz. geothermal energy utilization, polymerization, oil-reservoir, crude oil extraction, crystallization, etc.

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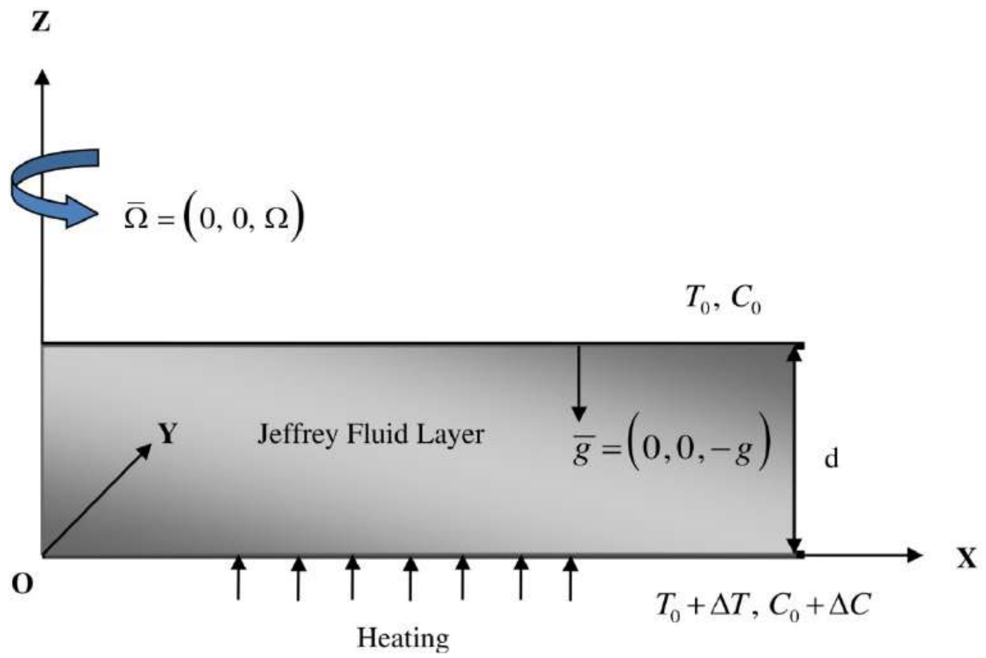

Figure 1. Schematic configuration of Jeffrey fluid layer.

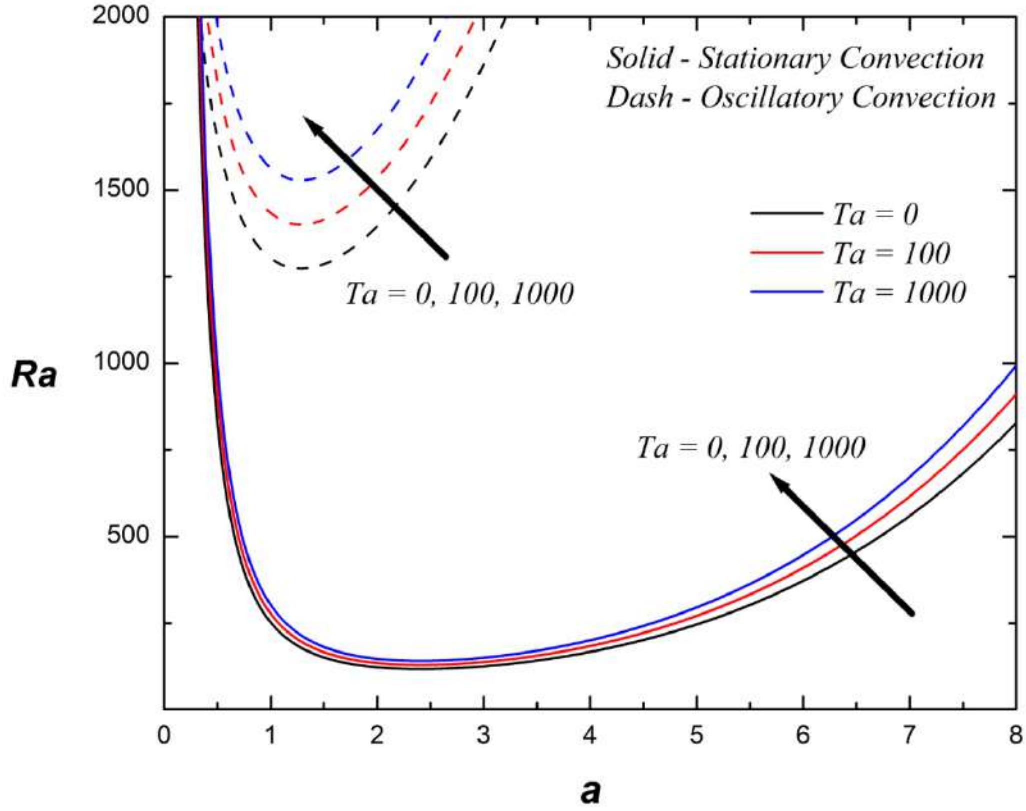

Figure 2. Effect of Rotational parameter viz. Taylor number (Ta) on Stability curves.

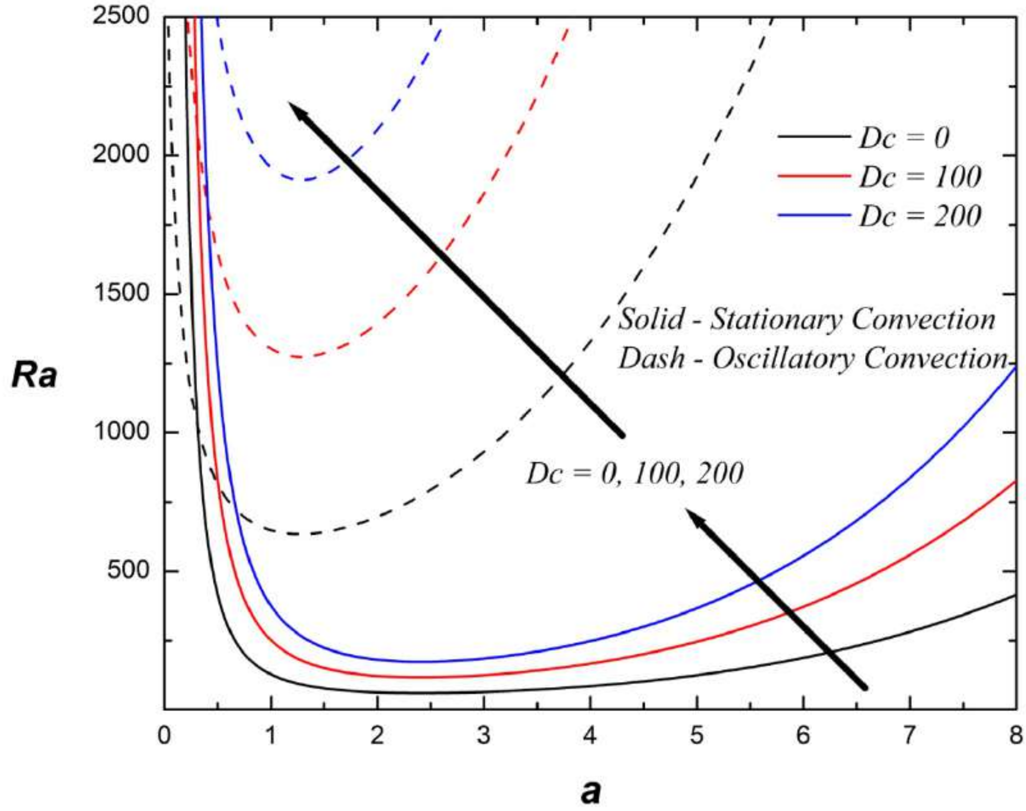

Figure 3. Effect of Darcy-Brinkman number (Dc) on Stability curves.

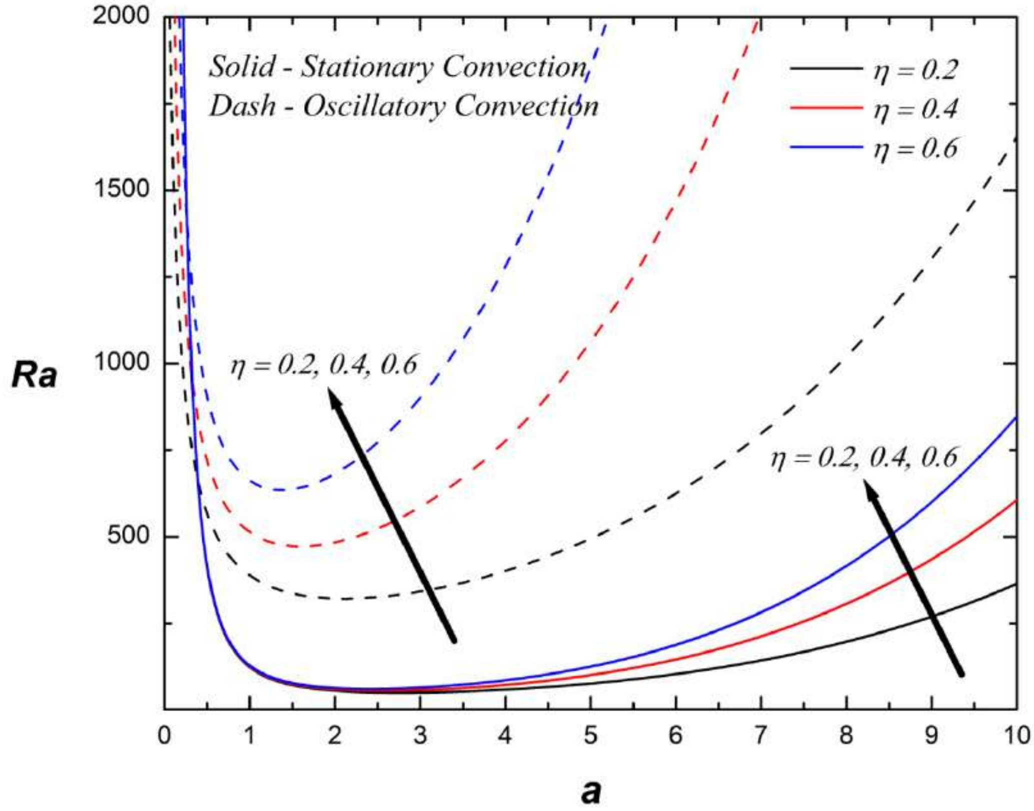

Figure 4. Effect of anisotropy in thermal diffusivity (*η)* on Stability curves.

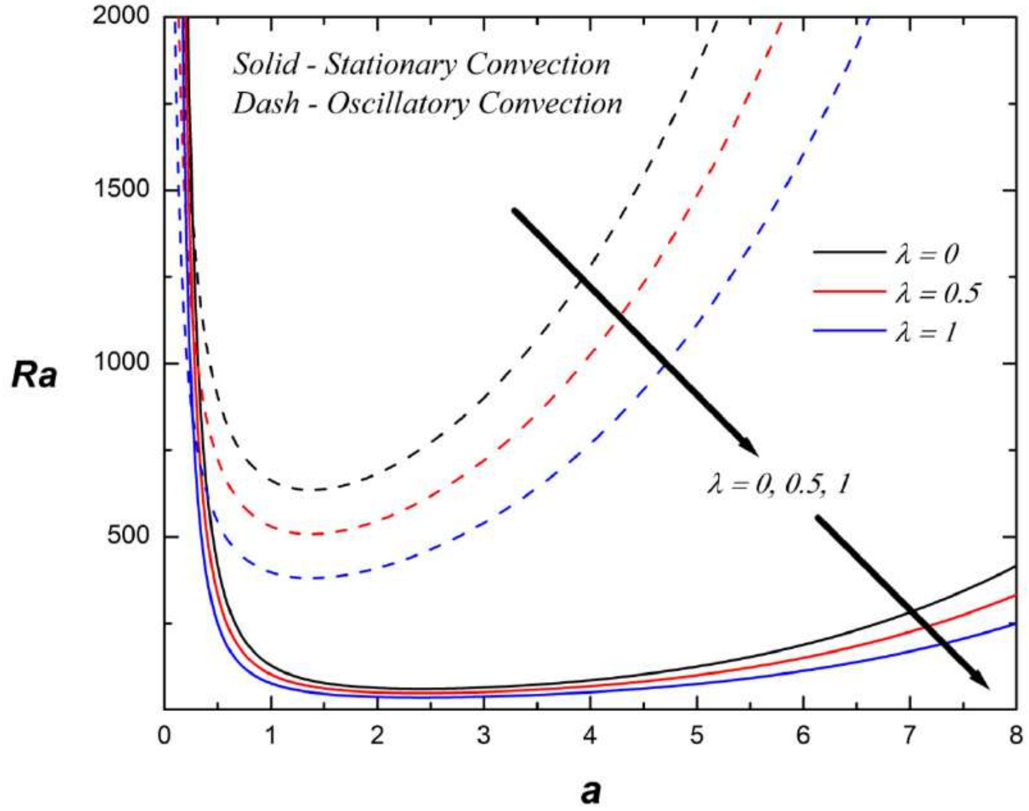

Figure 5. Effect of Jeffery fluid parameter (λ) on Stability curves.

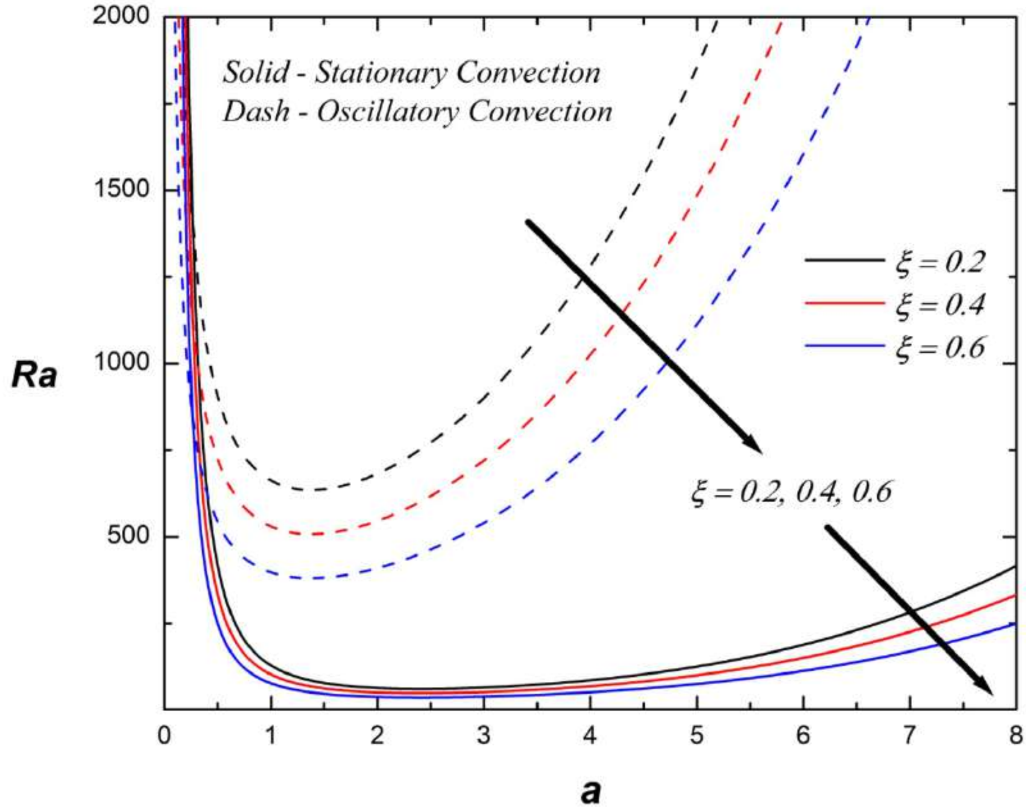

Figure 6. Effect of anisotropy in permeability (ξ) on Stability curves.

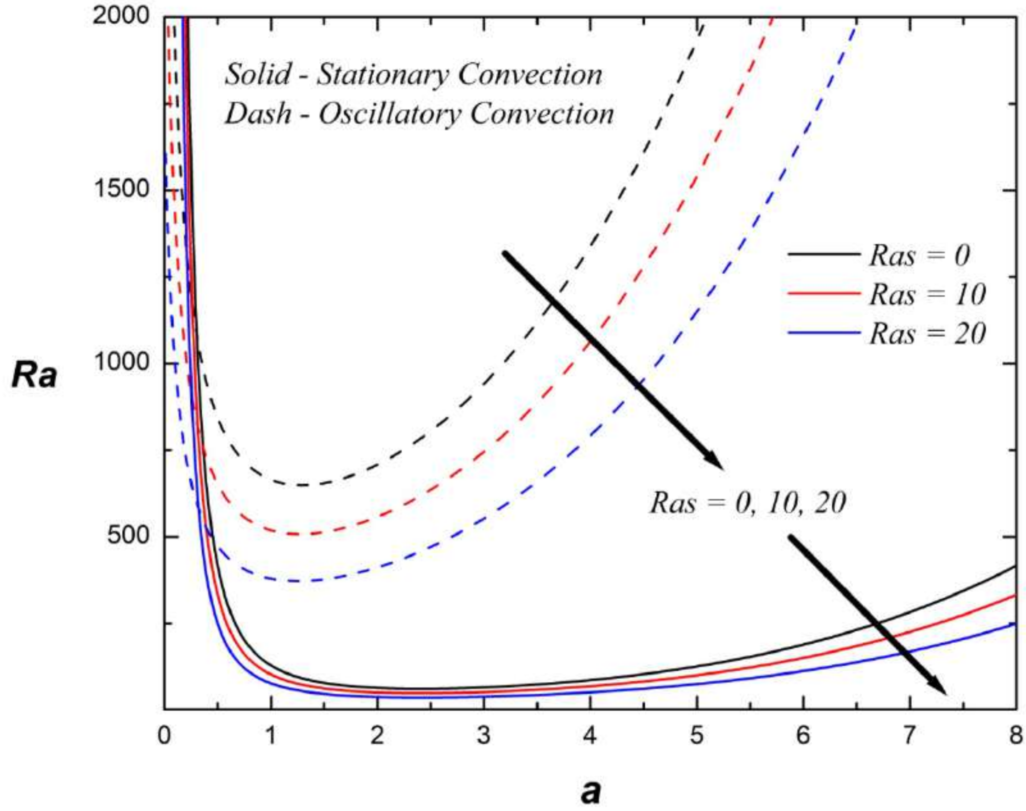

Figure 7. Effect of solute Rayleigh number (Ra_S) on Stability curves.

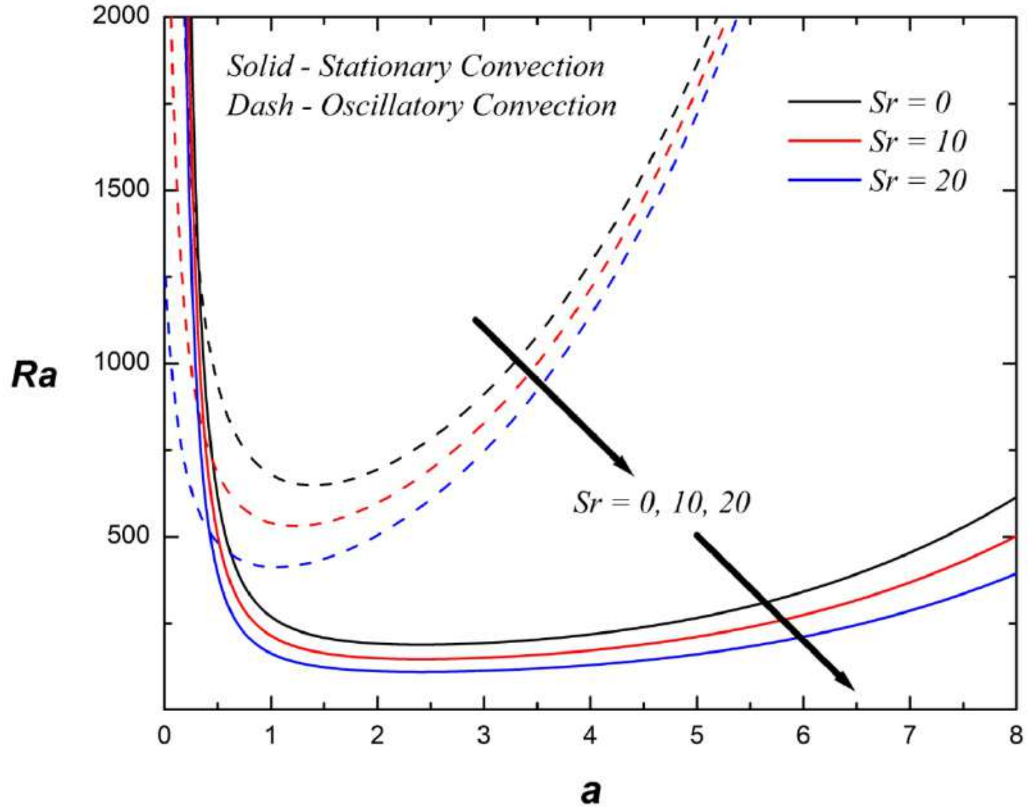

Figure 8. Effect of Soret parameter (*Sr*) on Stability curves.

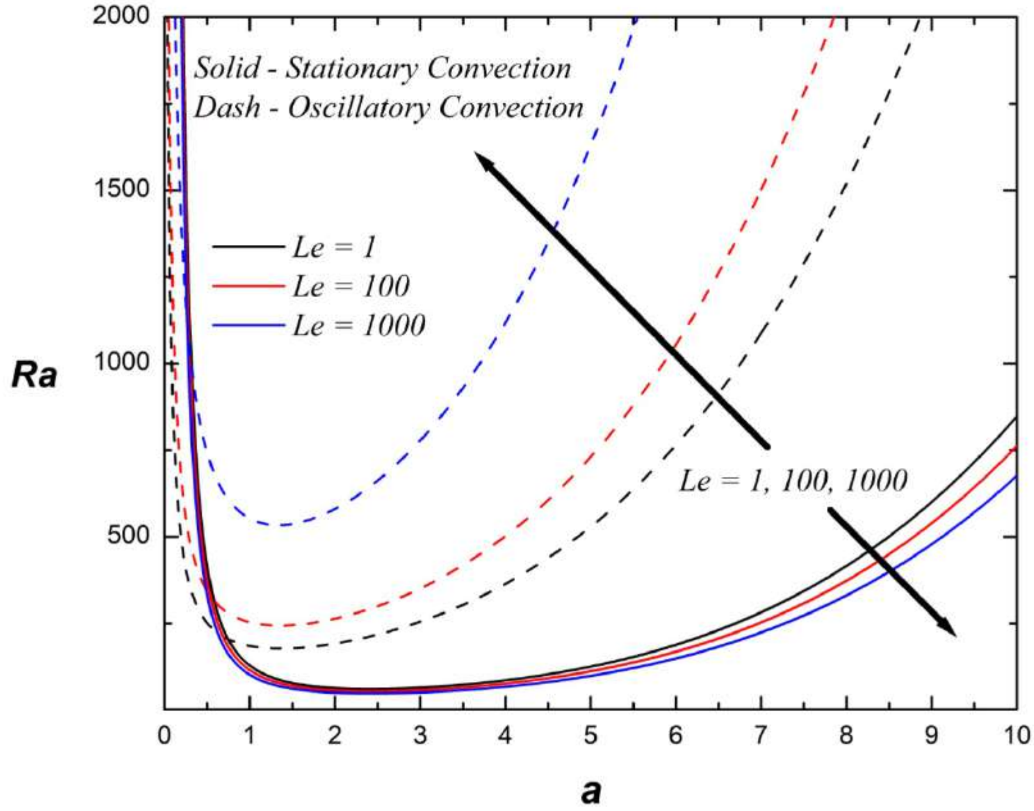

Figure 9. Effect of Lewis number (*Le*) on Stability curves.

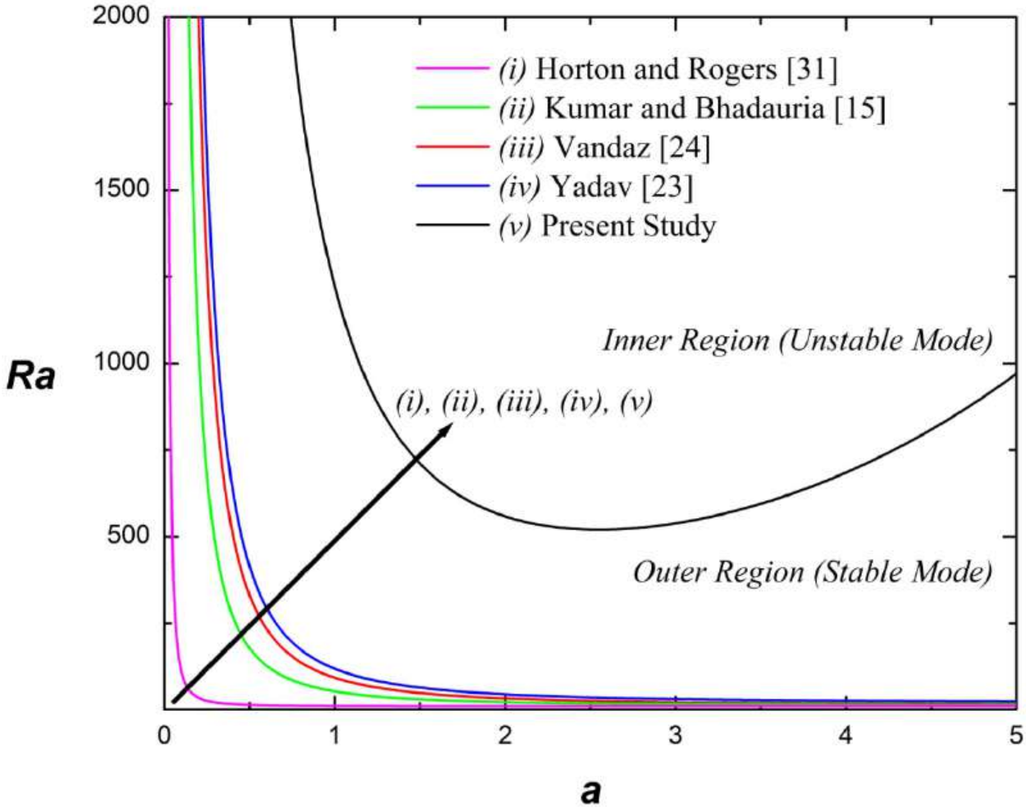

Figure 10. Comparative pictorial representation of Stability curves.