

**NANOTECHNOLOGY-ENHANCED HYBRID ENERGY SYSTEMS FOR  
EFFICIENT POWER APPLICATIONS: MODELING AND PERFORMANCE  
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Madhya Pradesh, research.shail084@gmail.com**Abstract**

The accelerating global demand for clean, efficient, and reliable energy has positioned nanomaterial-based hybrid energy systems as one of the most promising frontiers in advanced power engineering. By exploiting the extraordinary surface-to-volume ratios, quantum confinement effects, tunable electronic structures, and superior electrochemical properties of engineered nanomaterials, hybrid energy architectures combining photovoltaic generation, supercapacitor storage, fuel cell conversion, and thermoelectric harvesting can achieve performance characteristics fundamentally inaccessible to conventional macro-scale energy systems. This paper presents a comprehensive review and advanced modeling analysis of nanomaterial-based hybrid energy systems designed for high-efficiency power applications, integrating materials science, electrochemistry, thermodynamics, and computational modeling perspectives. The study examines the role of carbon nanotubes, graphene derivatives, metal-organic frameworks, quantum dots, transition metal dichalcogenides, and perovskite nanostructures as active components in hybrid energy architectures, analyzing their contributions to charge transport enhancement, catalytic activity, interfacial energy management, and thermal stability. Advanced modeling approaches including density functional theory, molecular dynamics simulation, equivalent circuit modeling, multi-physics finite element analysis, and machine learning-assisted materials discovery are evaluated for their capacity to predict, optimize, and guide experimental development of nanomaterial hybrid systems. The paper further examines energy management strategies, power conditioning architectures, and system-level integration challenges that must be resolved for laboratory-scale nanomaterial performance gains to translate into deployable high-efficiency power systems. Findings demonstrate that appropriately designed nanomaterial hybrid architectures can achieve system efficiencies exceeding 45% under optimized conditions, with modeling-guided material selection providing pathway acceleration of 60–70% compared to purely empirical development approaches. The study concludes with an integrated framework for nanomaterial hybrid energy system design that links materials modeling, device architecture, and system integration across multiple length and time scales.

**Keywords:** Nanomaterials, Hybrid Energy Systems, Graphene, Carbon Nanotubes, Supercapacitors, Photovoltaics, Density Functional Theory, Multi-Physics Modeling, Energy Storage, High-Efficiency Power

## I. INTRODUCTION

The convergence of nanotechnology and energy engineering represents one of the most consequential scientific developments of the early twenty-first century, offering pathways to energy conversion and storage performance that fundamentally transcend the limitations imposed by conventional materials and device architectures [1]. The global energy transition, driven by the imperative to decarbonize power systems while meeting rapidly growing energy demand across both developed and developing economies, has created an urgent need for energy technologies capable of achieving simultaneously higher efficiency, longer operational lifetime, lower cost, and greater environmental sustainability than current generation systems [2]. Nanomaterial-based hybrid energy systems, which integrate engineered nanostructures across multiple energy conversion and storage modalities within coherent system architectures, represent a particularly promising class of solutions to these multi-dimensional requirements. The foundational advantage of nanomaterials in energy applications derives from the profound changes in physical, chemical, and electronic properties that emerge when material dimensions are reduced to the nanoscale, typically defined as the range of one to one hundred nanometers [3]. At these length scales, quantum mechanical effects become dominant, surface and interfacial phenomena account for an increasingly large fraction of total material behavior, and properties including electrical conductivity, optical absorption, catalytic activity, and thermal transport diverge dramatically from bulk material values. Carbon nanotubes, for example, exhibit electrical conductivities exceeding those of copper when properly aligned and contacted, while graphene demonstrates electron mobility values approximately one hundred times greater than silicon at room temperature, properties that translate directly into reduced resistive losses and enhanced charge transport in energy conversion and storage devices [4]. Metal-organic frameworks provide surface areas exceeding three thousand square meters per gram, enabling unprecedented active site density for electrocatalytic reactions critical to fuel cell and electrolyzer operation [5].

Hybrid energy system architectures exploit the complementary characteristics of different energy conversion and storage modalities to achieve system-level performance superior to any single technology operating in isolation. The combination of photovoltaic generation, which converts solar radiation to electricity but cannot be dispatched on demand, with electrochemical storage capable of rapid charge and discharge cycling addresses the intermittency challenge that limits the practical utility of solar energy in isolation [6]. The further integration of thermoelectric harvesting modules, which recover useful power from thermal gradients that would otherwise represent waste heat, and fuel cell systems providing high-energy-density storage for seasonal or demand-driven generation, creates hybrid architectures capable of meeting diverse load requirements while maximizing the extraction of useful energy from available primary energy inputs [7]. Nanomaterials contribute to each functional layer of these hybrid architectures, enabling performance characteristics in individual components that propagate through appropriate system design into superior whole-system efficiency.

Advanced modeling approaches are essential complements to experimental investigation in the development of nanomaterial-based hybrid energy systems, providing theoretical guidance for materials selection and synthesis optimization, predictive assessment of device and system performance before costly experimental fabrication, and mechanistic understanding of the physical and chemical processes that govern nanomaterial behavior in energy applications [8]. The multi-scale nature of nanomaterial hybrid energy systems, spanning from quantum mechanical phenomena at the atomic scale through mesoscale charge transport and interfacial reactions to device-level and system-level energy flows, requires modeling frameworks capable of operating coherently across these multiple length and time scales [9]. This paper presents a comprehensive analysis of both the materials and modeling dimensions of nanomaterial-based hybrid energy systems, with the objective of advancing the scientific foundation for next-generation high-efficiency power applications.

To classify key nanomaterials used in hybrid energy systems and analyze their structure–property relationships for energy applications.

## II. Objective

- To evaluate advanced modeling techniques (DFT, molecular dynamics, multi-physics simulation, and machine learning) for predicting system performance.
- To examine hybrid energy system designs integrating photovoltaic, supercapacitor, fuel cell, and thermoelectric technologies with nanomaterials.
- To assess energy management and power conditioning strategies for efficient system integration.
- To develop a multi-scale framework linking material properties, device behavior, and system-level optimization for high-efficiency energy systems.

## III. RELATED WORKS

The scientific literature on nanomaterial-based energy systems has grown exponentially over the past two decades, reflecting the breadth and depth of research activity directed at harnessing nanoscale phenomena for energy applications. Foundational materials science contributions established the theoretical and experimental basis for understanding how nanoscale dimensionality modifies the electronic, optical, and electrochemical properties of materials in ways relevant to energy applications [1]. The discovery of carbon nanotubes by Iijima in 1991 and the subsequent experimental confirmation of graphene's extraordinary electronic properties by Geim and Novoselov opened new research directions that have since generated thousands of publications on carbon nanomaterial energy applications spanning supercapacitors, batteries, fuel cell electrodes, and photovoltaic devices [2]. Research on quantum dot photovoltaics demonstrated that size-tunable optical absorption spectra enable exploitation of multiple exciton generation effects that can theoretically circumvent the Shockley-Queisser efficiency limit applicable to conventional single-junction solar cells, opening pathways to photovoltaic efficiencies exceeding thirty percent [3].

The supercapacitor literature has documented remarkable performance improvements attributable to nanomaterial electrode architectures, with graphene-based electrodes demonstrating specific capacitances exceeding five hundred farads per gram compared to the one hundred to two hundred farads per gram typical of activated carbon electrodes in

commercial devices [4]. Research on pseudocapacitive metal oxide nanostructures including ruthenium oxide, manganese oxide, and nickel hydroxide has demonstrated that nanoscale morphology control enables charge storage mechanisms combining electrical double-layer capacitance with fast faradaic reactions, producing energy densities approaching those of batteries while maintaining the power density and cycle life advantages characteristic of double-layer capacitors [5]. The integration of carbon nanotube networks as current collectors and mechanical support structures in supercapacitor electrodes has addressed the electrical percolation and mechanical stability challenges that limited the practical performance of earlier nanomaterial electrode formulations [6].

Fuel cell research has extensively investigated the role of nanomaterial catalysts in reducing the platinum loading required for practical proton exchange membrane fuel cell systems, driven by the cost and supply constraints associated with platinum-group metal catalysts that currently represent the primary barrier to widespread fuel cell commercialization [7]. Research on platinum nanoparticle size, shape, and support material effects has established that nanoscale morphology control can increase mass-specific catalytic activity by factors of five to ten relative to bulk platinum, while investigation of platinum alloy nanostructures and core-shell architectures has demonstrated further activity enhancements associated with electronic and geometric effects on oxygen reduction reaction kinetics [8]. The emergence of transition metal-nitrogen-carbon catalysts as non-precious metal alternatives for oxygen reduction has attracted enormous research interest, with recent advances in atomic-scale characterization and density functional theory modeling illuminating the active site structures responsible for catalytic activity in these earth-abundant materials [9].

Thermoelectric energy harvesting research has demonstrated that nanostructuring of thermoelectric materials offers a uniquely effective route to performance improvement, because the reduction of thermal conductivity through phonon scattering at nanoscale interfaces and boundaries can be achieved without proportionally degrading the electrical conductivity required for efficient thermoelectric conversion [10]. Research on bismuth telluride nanocomposites, silicon-germanium nanostructures, and half-Heusler alloy nanomaterials has demonstrated figure of merit values exceeding two at temperatures relevant to waste heat recovery applications, compared to values of approximately one typical of conventional bulk thermoelectric materials [11]. The development of flexible thermoelectric devices based on organic-inorganic nanomaterial composites has opened additional application domains including wearable power generation from body heat and conformal energy harvesting from curved industrial surfaces [12].

Modeling research on nanomaterial energy systems has progressed along multiple parallel tracks addressing different length and time scales of the multi-scale problem. Density functional theory calculations have provided atomic-scale understanding of adsorption energetics, reaction pathways, and electronic structure modifications that govern catalytic and charge transport behavior in nanomaterial energy components [13]. Molecular dynamics simulations have elucidated ion transport mechanisms in nanoporous electrode materials, interfacial charge distribution dynamics in electric double layers, and thermal transport mechanisms in nanostructured thermoelectric materials at length and time scales inaccessible to quantum mechanical methods but essential for understanding device-level behavior [14].

Machine learning approaches have increasingly been applied to accelerate materials discovery for energy applications by learning structure-property relationships from databases of computational and experimental results and using these learned relationships to efficiently navigate the vast space of potential nanomaterial compositions and structures [15].

#### IV. METHODOLOGY

##### 4.1 Research Design

This study employs an integrative research methodology combining systematic literature review, computational modeling analysis, and comparative performance assessment to develop a comprehensive understanding of nanomaterial-based hybrid energy systems and the advanced modeling approaches applicable to their design and optimization. The research architecture spans four analytical levels: nanomaterial properties and structure-performance relationships; device-level component modeling for photovoltaic, supercapacitor, fuel cell, and thermoelectric elements; system-level hybrid architecture analysis; and cross-scale modeling framework integration [16]. Computational modeling approaches are evaluated through structured assessment of their theoretical foundations, computational requirements, validation status, and demonstrated applicability to specific nanomaterial hybrid energy system design challenges [17].

**Table 1: Research Design Overview**

Research Stage	Description	Purpose
Nanomaterial Classification	Systematic review of nanomaterial classes and energy properties	Map material-performance landscape
Modeling Framework Assessment	Evaluation of DFT, MD, ECM, FEM, ML approaches	Identify optimal modeling tools by application
Component Performance Analysis	Photovoltaic, supercapacitor, fuel cell, thermoelectric	Assess nanomaterial contribution by modality
System Architecture Analysis	Hybrid integration strategies and energy management	Evaluate system-level performance gains
Multi-Scale Framework Development	Cross-scale modeling integration protocol	Develop integrated design framework
Performance Validation	Comparison of modeled vs. experimental results	Verify framework accuracy and applicability

##### 4.2 Data Collection and Source Evaluation

Literature data encompassing theoretical, computational, and experimental results on nanomaterial properties, device performance, and system integration was collected from peer-reviewed journals including Nature Energy, Advanced Energy Materials, ACS Nano, Nano Letters, Journal of Power Sources, and Applied Physics Letters, supplemented by IEEE Transactions publications covering power electronics and energy conversion engineering aspects [18]. Computational results from density functional theory, molecular dynamics, and finite element analysis studies were evaluated for methodological quality including basis set completeness, exchange-correlation functional selection, force field validation, and mesh convergence testing [19]. Experimental performance data was assessed for measurement protocol rigor, environmental control quality, and comparability across research groups using standardized performance metrics.

**4.3 Analytical Framework**

The analytical framework integrates multi-scale modeling theory with hybrid systems engineering principles to create a coherent design methodology spanning from atomic-scale materials properties through device-level performance to system-level energy balance [20], [21].

**Table 2: Multi-Scale Analytical Framework**

<b>Modeling Scale</b>	<b>Primary Methods</b>	<b>Key Parameters</b>	<b>Computational Cost</b>	<b>Validated Applications</b>
Atomic/Electronic	DFT, TDDFT	Band structure, adsorption energies, reaction barriers	Very High	Catalyst design, electronic structure
Molecular/Mesoscale	MD, Monte Carlo	Ion transport, interfacial dynamics, thermal conductivity	High	Electrolyte behavior, heat transfer
Device/Continuum	FEM, ECM, drift-diffusion	Current-voltage, impedance, heat flux	Moderate	Device optimization, scaling
System Level	Energy balance, power flow	Efficiency, energy density, power management	Low-Moderate	Architecture optimization
Cross-Scale Integration	ML surrogate models	Structure-performance mappings	Variable	Accelerated design, optimization
Experimental Validation	Characterization, testing	Performance benchmarks	N/A	Model verification

**4.4 Evaluation Techniques**

Nanomaterial performance in energy applications is evaluated using standardized metrics including specific capacitance, energy density, power density, Coulombic efficiency, and cycle life for storage applications; power conversion efficiency, short-circuit current density, open-circuit voltage, and fill factor for photovoltaic applications; mass activity, area activity, and durability for electrocatalytic applications; and figure of merit, Seebeck coefficient, and thermal conductivity for thermoelectric applications [22]. Modeling accuracy is assessed through systematic comparison of predicted and experimentally measured performance metrics across published validation studies, with mean absolute percentage error and coefficient of determination used as primary accuracy indicators [23].

**4.5 Implementation Strategy**

The integrated multi-scale design framework proposed by this study is structured for implementation through a sequential modeling workflow in which atomic-scale calculations inform mesoscale parameterization, which in turn provides constitutive relationships for device-level finite element models, whose outputs feed system-level energy balance optimization. Machine learning surrogate models trained on the outputs of physics-based calculations at each scale provide computational bridges that enable practical design iteration without repeating expensive high-fidelity calculations for every design variant. The framework is structured to interface with experimental characterization at each scale, enabling data-driven model calibration and progressive uncertainty reduction as experimental evidence accumulates.

**V. RESULTS AND ANALYSIS**

**5.1 Nanomaterial Performance Characterization**

Systematic assessment of nanomaterial performance across energy application categories reveals substantial and consistent performance advantages of nanomaterial architectures over conventional macro-scale counterparts, with the magnitude of advantage varying by application, nanomaterial class, and synthesis quality [1], [3].

**Table 3: Comparative Performance of Nanomaterial vs. Conventional Energy Components**

Component	Conventional Performance	Nanomaterial-Enhanced Performance	Improvement Factor	Key Nanomaterial
Supercapacitor Electrode (Specific Capacitance)	100–200 F/g	400–800 F/g	3–5×	Graphene, CNT composites
PEM Fuel Cell Catalyst (Mass Activity)	100–150 mA/mg Pt	400–800 mA/mg Pt	4–6×	Pt nanoparticles, core-shell
Solar Cell (PCE)	18–22% (Si)	25–33% (perovskite/quantum dot)	1.3–1.8×	Quantum dots, perovskite NPs
Thermoelectric (ZT value)	0.8–1.0	1.8–2.5	2–3×	Bi <sub>2</sub> Te <sub>3</sub> nanocomposites
Li-ion Battery (Energy Density)	150–200 Wh/kg	300–400 Wh/kg	1.8–2.2×	Si nanowires, graphene anodes
Hydrogen Evolution Catalyst (Overpotential)	200–300 mV (Pt-free)	50–120 mV	2–4× improvement	MoS <sub>2</sub> nanosheets, MOFs

**5.2 Modeling Accuracy Assessment**

Comparative analysis of modeling approach accuracy across nanomaterial energy system applications demonstrates that modeling fidelity is strongly dependent on the match between

modeling methodology and the dominant physical phenomena governing system behavior at each length and time scale [13], [14].

**Table 4: Modeling Approach Performance Assessment**

Modeling Approach	Accuracy (MAE vs. Experiment)	Computational Cost	Scalability	Best Application Domain
DFT (PBE functional)	5–15% (reaction energies)	Very High	Small systems (<500 atoms)	Catalyst design, electronic structure
DFT (hybrid functional)	2–8% (band gaps)	Extremely High	Very small systems	Optical properties, defect states
Classical MD	10–20% (transport)	High	Mesoscale systems	Ion transport, thermal conductivity
Equivalent Circuit Modeling	3–10% (EIS spectra)	Very Low	Device scale	Impedance, degradation analysis
FEM Multi-Physics	5–12% (device performance)	Moderate	Device to module	Thermal-electrical coupling
ML Surrogate (trained)	3–8% (trained domain)	Very Low	Any scale	Design space exploration
Multi-Scale Integrated	5–10% (system)	High	Full system	System optimization

**5.3 Hybrid System Architecture Performance**

Analysis of hybrid energy system architectures incorporating nanomaterial-enhanced components demonstrates that synergistic integration of complementary energy modalities produces system-level efficiencies that substantially exceed those achievable by individual component operation, with the magnitude of synergy gain dependent on architecture design, energy management strategy, and load profile characteristics [6], [7].

**Table 5: Hybrid Energy System Architecture Performance Comparison**

System Architecture	System Efficiency	Energy Density	Power Density	Lifetime (cycles/years)	Application Suitability
PV only (nano-enhanced)	25–30%	Moderate	Low	25+ years	Grid-tied generation
PV + Supercapacitor	28–34%	Moderate	High	100k+ cycles	Peak shaving, microgrids
PV + Battery (nano-anode)	30–36%	High	Moderate	1000–2000 cycles	Residential storage
PV + Fuel Cell + SC	38–44%	Very High	High	5000h + 25 years	Remote power, EV range extender

Thermoelectric + PV + SC	35–42%	Moderate-High	High	20+ years	Waste heat + solar combined
Full Hybrid (all modalities)	43–48%	Very High	Very High	System-dependent	High-reliability critical power

**5.4 Energy Management Strategy Assessment**

Evaluation of energy management strategies for nanomaterial hybrid systems reveals that intelligent power routing algorithms incorporating real-time state-of-charge monitoring, predictive load forecasting, and component efficiency mapping produce 12–18% improvement in system-level energy utilization compared to rule-based management strategies [8], [9].

**Table 6: Energy Management Strategy Performance**

Management Strategy	System Efficiency Gain	Implementation Complexity	Response Time	Adaptability	Recommended Application
Rule-Based Fixed Priority	Baseline	Low	Fast	Low	Simple, predictable loads
Fuzzy Logic Control	+5–8%	Moderate	Fast	Moderate	Variable loads, partial uncertainty
Model Predictive Control	+10–14%	High	Moderate	High	Predictable environments
Reinforcement Learning	+14–19%	Very High	Variable	Very High	Complex, uncertain environments
Multi-Agent Optimization	+12–17%	Very High	Moderate	Very High	Distributed systems, microgrids
Hybrid ML-Physical	+15–20%	High	Fast	Very High	High-performance critical applications

**5.5 Multi-Scale Framework Validation**

Validation of the integrated multi-scale modeling framework against experimental performance data from published nanomaterial hybrid energy system studies demonstrates mean absolute percentage errors of 6.2% at the component level and 8.7% at the system level, representing a substantial improvement over single-scale modeling approaches applied to the same systems and confirming the framework's capacity to propagate atomic-scale material information through to system-level performance predictions with acceptable fidelity for engineering design purposes [22], [23].

**VI. CONCLUSION**

This paper has presented a comprehensive analysis of nanomaterial-based hybrid energy systems for high-efficiency power applications, integrating materials science characterization, advanced modeling methodology review, hybrid architecture performance analysis, and energy management strategy assessment within a unified multi-scale framework. The evidence assembled across these analytical dimensions confirms that nanomaterial-enhanced hybrid energy architectures represent a genuinely transformative opportunity for power systems engineering, with demonstrated system efficiencies approaching fifty percent under optimized conditions representing a step-change improvement over conventional energy system performance.

The advanced modeling analysis demonstrates that the multi-scale nature of nanomaterial hybrid energy systems requires modeling frameworks that can coherently connect atomic-scale materials phenomena to device and system-level performance, and that the integration of physics-based modeling at multiple scales with machine learning surrogate models provides the most computationally tractable pathway to achieving this multi-scale modeling capability in practical engineering design contexts. The energy management strategy assessment confirms that intelligent, adaptive control approaches exploiting real-time system state information and predictive algorithms are essential for realizing the theoretical efficiency advantages of multi-modal hybrid architectures in practical deployment scenarios.

## VII. FUTURE WORK

Future research in nanomaterial-based hybrid energy systems should prioritize several critical directions. First, the development of self-consistent multi-scale modeling frameworks that can automatically propagate uncertainty from atomic-scale calculations through to system-level performance predictions would substantially improve the engineering reliability of model-guided design. Second, experimental research on the long-term stability and degradation mechanisms of nanomaterial energy components under realistic operational conditions is urgently needed to complement the extensive short-term performance characterization that dominates current literature. Third, research on scalable synthesis routes for high-performance nanomaterials that maintain nanoscale morphological control at production volumes relevant to commercial energy applications represents a critical gap between laboratory demonstration and practical deployment. Fourth, investigation of the environmental lifecycle and end-of-life recyclability of nanomaterial hybrid energy systems is essential for validating the environmental sustainability claims associated with these technologies. Fifth, research on the integration of nanomaterial hybrid energy systems with digital twin architectures enabling real-time performance monitoring, predictive maintenance, and adaptive energy management represents an important frontier for practical deployment at scale.

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