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# THE REGULAR NUMBER OF A STRONGLY REGULAR FERRERS GRAPH

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## Abstract

The regular number r(G) of G is the minimum number of subsets into which the edge set of G is partitioned so that the subgraph induced by each subset is regular. In this article, we define the new concept Strongly regular ferrers graph and we investigate r(G) of Strongly regular ferrers graph.

**AMS Subject Classification:** 05C50, 05C25.

**Keywords:**Ferrers graph, Strongly regular graph, Strongly regular ferrers graph, regular number.

## 1.Introduction

In this article, we study a special kind of graph, namely Ferrers graph, which has practical applications in communication networks. The Ferrers relation was introduced for the first time [12] and has been utilized for various purposes across extensive scientific fields. The relation was employed with concept lattices in formal concept analysis [12]. Some graphs associated with the relation were again linked to concept lattices [12]. A kind of partition is presented as Ferrers diagrams. Preference modeling structures were constructed using generalized version of the relation in Social Choice Theory. In this study, we introduce a new graph class called Ferrers-esque graphs, defined by the relation. We provide a characterization of the class to determine whether an arbitrary simple graph is in the class or not – in other words, whether the graph is a Ferrers-esque graph. 'Ferrers' is used as an abbreviation for 'Ferrers-esque graph.

A simple graph G is a Ferrers graph if for all distinct x, y, z, w $\in$V(G) if xy $\in$ E(G) and zw$\in$ E(G) then either xw $\in$ E(G) or yz $\in$ E(G). Since xy $\in$ E(G) $\Leftrightarrow$ yx $\in$ E(G) holds for all simple graphs, the definition of Ferrers graph must be extended to if xy $\in$ E(G) and zw $\in$ E(G) then either
xw $\in$ E(G) or yz $\in$ E(G) or yw $\in$ E(G) or xz $\in$ E(G). Moreover, in this article, we explore a new parameter known as the regular number of a graph. The definition of the regular number was introduced by V.R. Kuli, B. Janakiram, and Radha R. Iyer [9]. The regular number of a graph G, denoted as r(G), is the minimum number of subsets into which the edge set of G should be partitioned so that the subgraph induced by each subset is regular.

A strongly regular graph is a type of combinatorial object in graph theory with specific structural properties. It is characterized by three parameters: the number of vertices (n), the valency (k), and the two eigenvalues (λ and μ). A graph G of order n is called strongly regular with parameters (n, k, λ, μ) if:

(i) Every pair of adjacent vertices has a common number of neighbors (regularity): If two vertices are adjacent, they share exactly λ common neighbors.

(ii) Every pair of non-adjacent vertices has a common number of non-neighbors (strong regularity): If two vertices are not adjacent, they share exactly μ common non-neighbors.

(iii) The graph is regular: Every vertex has the same degree, which is k.

It is observed that, for $1 \leq k < n - 1$, the complete graph and the null graph of n vertices are not classified as strongly regular. Strongly regular graphs are interesting because they provide a balance between regularity and irregularity, making them a useful concept in coding theory, design theory, and algebraic graph theory. Strongly regular graphs often exhibit symmetry properties. They may be symmetric with respect to their center, reflecting a balanced structure. Strongly regular graphs are distance-regular, meaning that the number of paths of a given length between any two vertices depends only on the distance between them. Strongly regular graphs may have a significant number of automorphisms, reflecting symmetries and preserving adjacency relationships.

In this article, we introduce the new concept namely Strongly Regular Ferrers graphs and discuss some properties associated with them. Moreover, it is easily identified that, the number r(G) of a strongly regular Ferrers graph consistently holds a specific value and is always one.

**Definition 1.1.[8].** The eccentricity ece(v) or e(v) of a vertex v is the distance to a farthest vertex from v. A vertex v is said to be an ecentric vertex of u if deg(u,v) =ece(v).

Theorem 1.2. Let $G = C_n$ be a cycle graph. Then $C_n$ is a Ferrers graph for n = 4, 5 and a non-Ferrers graph otherwise.

**Theorem 1.3.[11].** For any graph G, r(G) = 1 if and only if G is regular.

**Theorem 1.4.** For a cycle $C_n$, where $n \geq 5$, the complement graph $\overline{C_n}$, is a Ferrers graph.

## 2. Strongly Regular Ferrers graph

The strongly regular graph was introduced by R.C.Bose in [9]. We incorporate the same concept in Ferrers graphs. It is defined as follows:

**Definition 2.1.** A graph G of order n is called a Strongly Regular Ferrers graph with parameters (n, k, λ, μ) if it satisfy the following conditions,

(i) G is a strongly regular graph

(ii) G is a Ferrers graph

**Example 2.2.** Consider the graph given in Figure 2.1. It is evident that G satisfies both conditions required for a strongly regular Ferrers graph. Consequently, G is called a strongly regular Ferrers graph with parameters (6, 4, 3, 4).

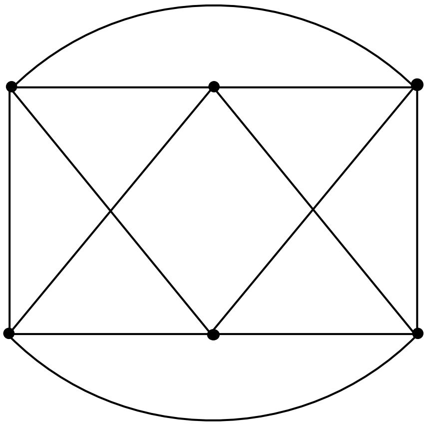

**Figure 5.1 Strongly Regular Ferrers graph**

**Observation 2.3.** The Complete graph $G = K_n$ is not a strongly regular Ferrers graph. Since the conditions of strongly regular graphs are not satisfied, the complete graph does not qualify as a strongly regular Ferrers graph.

**Theorem 2.4.** Let $G = C_n$ be a cycle. Then $C_n$ is a strongly regular graph for n = 4, 5 and not strongly regular otherwise.

**Proof.** Let $G = C_n$ be a cycle with n vertices.

**Case (i)** n = 3

By the definition of strongly regular, $G = C_3$ is not a strongly regular graph.

**Case (ii)** n = 4

Let $C_4 = v_1e_1v_2e_2v_3e_3v_4e_4v_1$ be the cycle with degree 2. To find the number of common neighbours in adjacent and non-adjacent vertices. First, we identify the neighbours of each vertex in $C_4$. Neighbours of the vertex $v_1$ is $v_2, v_4$ ; neighbours of the vertex $v_2$ is $v_1, v_3$; neighbours of the vertex $v_3$ is $v_2, v_4$; neighbours of the vertex $v_4$ is $v_1, v_3$. Then the number of common neighbours in the adjacent vertices is 0, and the number of common neighbours in non-adjacent vertices is 2. Therefore, by the definition of strongly regular, $C_4$ is strongly regular with parameters (4, 2, 0, 2).

**Case (iii)** When n = 5

Let $C_5 = v_1e_1v_2e_2v_3e_3v_4e_4v_5e_5v_1$ be the cycle. To find the neighbours of of each vertex in $C_5$. Clearly, $v_1$ is adjacent to $v_2$ and $v_5$ , $v_2$ is adjacent to $v_1$ and $v_3$ , $v_3$ is adjacent to $v_2$ and $v_4$, $v_4$ is adjacent to $v_3$ and $v_5$, $v_5$ is adjacent to $v_4$ and $v_1$. The number of common neighbours in the adjacent vertices of G is 0, and the number of common neighbours in the non-adjacent vertices of G is 1. Therefore, by the definition of strongly regular, $C_5$ is strongly regular with parameters (5, 2, 0, 1).

**Case (iv)** When $n \geq 6$

Let $G = C_n$ where $n \geq 6$. $G = C_n$ where $n \geq 6$ is a 2-regular graph. However, a pair of non-adjacent vertices may have either 1 common neighbor or none. Hence $G = C_n$ with $n \geq 6$ is not strongly regular.

**Theorem 2.5.** The cycle, $G = C_n$ is a strongly regular Ferrers graph if and only if n is either 4 or 5.

**Proof.** Let $G = C_n$ be a cycle with n vertices.

**Case (i)** $n \leq 3$

For $n = 1, 2, 3$, we have $C_1$, $C_2$, $C_3$ are excluded since they are complete graphs. As established, complete graphs are not regarded as strongly regular.

**Case (ii)** $n = 4$

We know that every cycle graph is 2 regular. Since $G = C_4$, by Theorem 1.2, $C_4$ is a Ferrers graph. Furthermore, based on Theorem 2.4, $C_4$ is a strongly regular graph. Therefore, by the definition 2.1, $C_4$ is identified as a strongly regular Ferrers graph.

**Case (iii)** $n = 5$

This case parallels Case (ii).

**Case (iv)** $n \geq 6$

Consider the cycle graph $C_n$, denoted as $C_n = v_1e_1v_2e_2 \ldots e_{i-1}v_ie_iv_{i+1} \ldots e_{n-1}v_ne_nv_1$. According to Theorem 1.2, $G = C_n$, where $n \geq 6$ is classified as a non-Ferrers graph. Consequently, based on the definition provided in 2.1, $G = C_n$ where $n \geq 6$ is not a strongly regular Ferrers graph.

**Theorem 2.6.** For a cycle $C_n$, $\overline{C_n}$ is a strongly regular graph for $n = 5$ and not-strongly regular otherwise.

**Proof.** Let $C_n$ be any cycle with n vertices and let $\overline{C_n}$ be the complement of the graph $C_n$.

**Case (i)** $n = 3$

For $n = 3$, $\overline{C_3}$ is a null graph, and as stated, null graphs are not considered strongly regular.

**Case (ii)** $n = 4$

For $n = 4$, $\overline{C_4}$ is a disconnected graph. Disconnected graphs are not strongly regular.

**Case (iii)** $n = 5$

Let $C_5 = v_1e_1v_2e_2v_3e_3v_4e_4v_5e_5v_1$ be the cycle. Then $\overline{C_5} = \overline{v_1e_1v_3e_2v_5e_3v_2e_4v_4e_5v_1}$ is also a cycle. Clearly, $\overline{C_5}$ is a 2-regular graph. To identify the number of common neighbours of adjacent and non-adjacent vertices. Now, $\overline{v_1}$is adjacent to $\overline{v_3}, \overline{v_4}$ ; $\overline{v_2}$is adjacent to $\overline{v_4}, \overline{v_5}$ ; $\overline{v_3}$ is adjacent to $\overline{v_1}, \overline{v_5}$ ; $\overline{v_4}$ is adjacent to $\overline{v_1}, \overline{v_2}$; $\overline{v_5}$ is adjacent to $\overline{v_2}, \overline{v_3}$ . Thus, the number of common neighbours in the adjacent vertices is 0 and the number of common neighbours in the non-adjacent vertices is 1. Therefore, by the definition of strongly regular, $\overline{C_5}$ is strongly regular with parameters (5, 2, 0, 1).

**Case (iv)** $n \geq 6$

For $n \geq 6$, $\overline{C_n}$ is an $n - 3$ regular graph because, in the complement graph, a vertex is connected to every other vertex except itself and its immediate neighbors in the cycle. But a pair of non-adjacent vertices may have either 1 common neighbour or none. Let's analyze the possible number of common neighbors for a pair of non-adjacent vertices. Consider two non adjacent vertices $v_i$ and $v_j$ in $C_n$. Without loss of generality, let $i < j$ (i.e., $v_i$ comes before $v_j$ in the cycle). Now, $\overline{C_n}$ has an edge between $v_i$ and $v_j$ if and only if there is no edge between $v_i$ and $v_j$ in $C_n$. In $C_n$, $v_i$ and $v_j$ are non-adjacent, and since $C_n$ is a cycle, the distance between $v_i$ and $v_j$ is $\frac{n}{2}$ in terms of the number of vertices. If n is even, $\frac{n}{2}$ is an integer, and there is an even number of vertices between $v_i$ and $v_j$ in the cycle. Therefore, there are $\frac{n}{2} - 1$ vertices between $v_i$ and $v_j$ that are not adjacent to either $v_i$ or $v_j$. If n is odd, $\frac{n}{2}$ is not an integer, but $\lfloor \frac{n}{2} \rfloor$ is, and there are $\lfloor \frac{n}{2} \rfloor$ vertices between $v_i$ and $v_j$ that are not adjacent to either $v_i$ or $v_j$. In either case, there are at least $\lfloor \frac{n}{2} \rfloor$ vertices between $v_i$ and $v_j$ that are not adjacent to either $v_i$ or $v_j$. Since $\overline{C_n}$ is stated to be an n −3 regular graph, each vertex in $\overline{C_n}$ has degree n −3. This means that each vertex in $\overline{C_n}$ is adjacent to n −3 other vertices. Therefore, for n ≥ 6, $\overline{C_n}$ has at least $\lfloor\frac{n}{2}\rfloor$ common neighbors for any pair of non-adjacent vertices, which can be either 1 or more. Therefore, $\overline{C_n}$ is an n −3 regular graph for n ≥ 6.

**Theorem 2.7.** For a cycle $C_n$, $\overline{C_n}$ is a strongly regular Ferrers graph for n = 5 and not a strongly regular Ferrers graph otherwise.

**Proof.** Let $C_n$ be any cycle with n vertices. And let $\overline{C_n}$ denote the complement of the graph $C_n$.

**Case (i)** n ≤ 4

For n = 3, $\overline{C_3}$ is a null graph, which is not strongly regular and not a Ferrers graph.

For n = 4, $\overline{C_4}$ is a disconnected graph, which is not considered a Ferrers graph and not strongly regular.

**Case (ii)** n = 5

For n = 5, $\overline{C_5}$ is a 2-regular graph. According to the Theorem 2.6, $\overline{C_5}$ is strongly regular. Also, By Theorem 1.4, $\overline{C_5}$ is a Ferrers graph. Hence, $\overline{C_5}$ is a strongly regular Ferrers graph.

**Case (iii)** n ≥ 6

By Theorem 2.6, $\overline{C_n}$ when n ≥ 6 is not strongly regular. Hence, $\overline{C_n}$ is not strongly regular Ferrers graph.

**Theorem 2.8.** Every connected strongly regular graph has diameter 2 and radius 2.

**Proof.** Let G be a strongly regular graph with n vertices with k neighbours. To prove the diameter of G is 2. Let u be any vertex of G. Vertex u is adjacent to some $x_i$ and not adjacent to some $y_i$ where $x_i, y_i \in G$ and $x_i$ is adjacent to $y_i$.. By definition 1.1, the eccentricity of u is 2. Therefore, the diameter of G is 2. Now, to prove the radius of G is 2. We know that, $\mathrm{rad}(G) \leq \mathrm{diam}(G)$. Therefore, rad(G) is either 2 or 1. Suppose $\mathrm{rad}(G) = 1$, then G is either a complete graph or a star graph or $C_3$ or $P_2$ or $P_3$. In all these cases G is not a strongly regular graph, which is a contradiction. Hence rad(G) = 2.

**Theorem 2.9.** Every connected strongly regular Ferrers graph has diameter 2 and radius 2.

**Proof.** Let G be a strongly regular Ferrers graph. The goal is to establish that diam(G) = 2 and rad(G) = 2. According to the definition 2.1, Every strongly regular Ferrers graph is also strongly regular. By Theorem 2.8, it follows that $\mathrm{diam}(G) = 2$ and $\mathrm{rad}(G) = 2$.

**Theorem 2.10.** Let G be a strongly regular Ferrers graph with parameters (n, k, λ, μ). Then $k(k - \lambda - 1) = (n - k - 1)\mu$.

**Proof.** Let G be a strongly regular Ferrers graph with parameters (n, k, λ, μ). The objective is to prove that, $k(k - \lambda - 1) = (n - k - 1)\mu$. We count the number of neighbours in two ways: by considering the number of ordered triples of the form (u, v, w) in G with the property that v is adjacent to both u and w and that u and w are not adjacent to each other. Suppose we choose v first. We have n choices for v, and then the number of choices available for a neighbor u of v is k. Having chosen u, the final step is to choose a vertex w that is adjacent to v but is not a common neighbour of u and v. There are $k - 1$ neighbours of v from which w may be chosen, but λ of these are also neighbours of u. So, the number of choices for w is k-1-λ. Hence, the number of choices for the triple (u, v, w) is $nk(k - \lambda - 1)$. On the other hand, suppose we choose u first. We have n choices for u, and then we may choose w from among the n – k –1 non-neighbours of u. Having done this, we have μ choices for v among the common neighbours of u and w.

So the number of choices for the triple (u, v, w) is n(n-k-1) $\mu$. Putting these two counts together we find, $nk(k-\lambda-1) = n(n-k-1)\mu$. This implies that, $k(k-\lambda-1) = (n-k-1)\mu$.

**Remark 2.11.** Let G be a strongly regular Ferrers graph with parameters (n, k, $\lambda$, $\mu$). Then the numbers f and $g = \frac{1}{2}\left(n-1 \pm \frac{(n-1)(\mu-\lambda)-2k}{\sqrt{(\mu-\lambda)^2+4(k-\mu)}}\right)$ are non-negative integers.

**Example 2.12.** AL($C_6$) is a strongly regular Ferrers graph with parameters (6, 4, 2, 4). Then, by the above Remark 2.11, we get, f = 3 and g = 2. Here f and g are non – negative integers. **Theorem 2.13.** Let G be a strongly regular Ferrers graph with parameters (n, k, $\lambda$, $\mu$). Then $\overline{G}$ is a strongly regular graph and may or may not be Ferrers with parameters ($n, n-k-1, n-2-2k+\mu, n-2k+\lambda$).

**Proof.** Let uv be an edge of $\overline{G}$. The number of triangles to which uv belongs in $\overline{G}$ is the number of vertices in G that are adjacent to neither u nor v. In G, uv is not an edge, each of u and v has k neighbours, and $\mu$ vertices are common neighbours of u and v. So, the number of vertices that are adjacent to atleast one of u and v is $2k-\mu$. Thus, the number of vertices (other than u and v) that is adjacent to neither u nor v is $n-2-2k+\mu$. This is the number of triangles to which the edge uv belongs in $\overline{G}$. Now suppose that u and v are non-adjacent edges in $\overline{G}$. Then uv is an edge of G. Each of u and v has $k-1$ additional neighbours in G, and they have $\lambda$ common neighbours, so $2k-2-\lambda$ is the number of vertices (other than u and v themselves) that are adjacent in G to at least one of u and v. That leaves $n-2-(2k-2-\lambda)$ or $n-2k+\lambda$ vertices in G that are adjacent to neither u nor v. This is the number of common neighbours of u and v in $\bar{G}$.Therefore, we conclude that $\bar{G}$ is a strongly regular graph with parameters ($n, n-k-1, n-2-2k+\mu, n-2k+\lambda$).

**Remark 2.14.** (i) Petersen graph is the example for the complement of a strongly regular graph is strongly regular.

(ii) $C_4$ is the complement of a strongly regular Ferrers graph is strongly regular but may or may not be a Ferrers graph.

## 3. The Regular Number of Strongly Regular Ferrers Graph

**Theorem 3.1.** The regular number of every strongly regular Ferrers graph is one.

**Proof.** Let G be any strongly regular Ferrers graph. We know that, every strongly regular Ferrers graph is regular. Therefore, By Theorem 1.3, $r(G) = 1$.

## 4. Conclusion

In this article, we found the definition of Strongly regular ferrers graph and the number r(G) of a strongly regular Ferrers graph consistently holds a specific value and is always one.

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